

# Advanced soil models. Calibration of Modified ECP-Hujeux model

©ZACE Services Ltd

21.08.2007

# Introduction

- The multi-mechanism elasto-plastic model (Aubry 1982, Hujeux 1985) is designed to properly reproduce soil behavior subject to monotonic and cyclic loads
- The original version of the model consists of three plane-strain deviatoric plastic mechanisms in three orthogonal planes and a single volumetric mechanism
- Hardening laws for deviatoric mechanisms: mixed isotropic/kinematic type
- Major extension of the basic Aubry's model, made by Hujeux, is related to the introduction of so-called domains of soil behavior for the three deviatoric mechanisms
  - Linear domain:  $0 \leq \gamma_c \leq 10^{-5}$
  - Stabilized hysteretic domain:  $10^{-5} \leq \gamma_c \leq 10^{-4}$
  - Nonstabilized hysteretic domain:  $10^{-4} \leq \gamma_c \leq 5 \times 10^{-3}$
  - Domain of incremental laws:  $\gamma_c \geq 5 \times 10^{-3}$

- In the current version of Z\_Soil the modified version of the model is implemented
- 3 deviatoric yield surfaces are replaced by one which includes an effect of strength anisotropy in the deviatoric plane
- Cyclic mechanisms are not included

# Notation: Stress and strain quantities

- $p = -\frac{1}{3}\sigma_{kk}$
- $s_{ij} = \sigma_{ij} + p\delta_{ij}$
- $q = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{\frac{1}{2}}$
- $\varepsilon_v = \varepsilon_{kk}$
- $e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_v\delta_{ij}$
- $\dot{\varepsilon}_D = \left(\frac{2}{3}\dot{e}_{ij}\dot{e}_{ij}\right)^{\frac{1}{2}}$

NB. Indices  $i, j, k$  vary from 1 to 3

# Nonlinear elastic behavior

The recoverable part of the deformation is driven by the initial shear  $G$  and bulk  $K$  moduli expressed by the following power laws:

$$G = G_a \left( \frac{p}{p_{\text{ref}}} \right)^n \quad K = K_a \left( \frac{p}{p_{\text{ref}}} \right)^n$$

$$\frac{G}{K} = \frac{G_a}{K_a} = \text{const} = \frac{3}{2} \frac{1 - 2\nu}{1 + \nu}$$

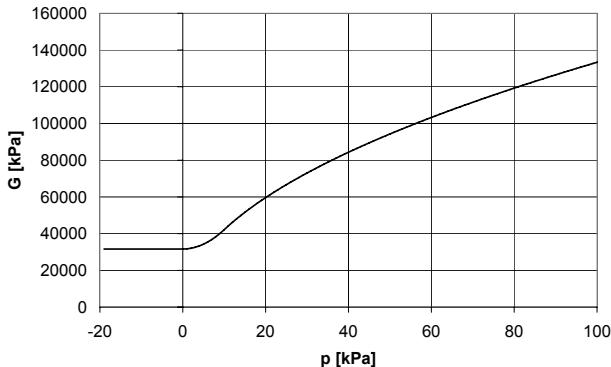
From the algorithmic point of view it may happen that  $p$  can be negative, hence the following regularization is used:

$$\begin{aligned} G &= G_a \left( \frac{p}{p_{\text{ref}}} \right)^n & \text{if } p \geq p_L \\ G &= \bar{G} + A p^2 & \text{if } 0 \leq p < p_L \\ G &= \bar{G} & \text{if } p < 0 \end{aligned}$$

# Nonlinear elastic behavior

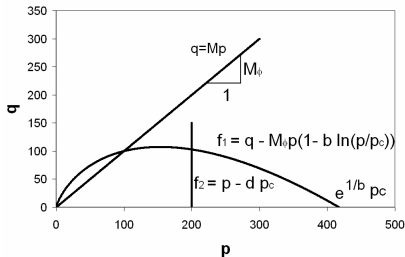
Example of  $G(p)$  function

( $G_a = 10000$  kPa,  $p_{\text{ref}} = 1$  kPa,  $n = 0.5$ ,  $p_L = 10$  kPa)

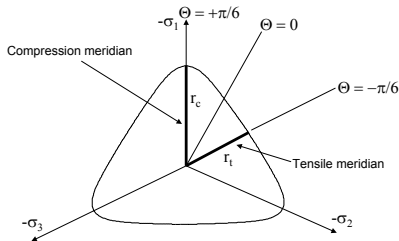


NB. This regularization preserves continuity of the  $G(p)$  formula up to first derivative which is crucial for the implicit implementation

# Yield surfaces



(a) Section by p-q plane



(b) Section by deviatoric plane

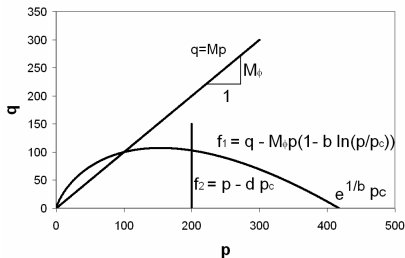
$$f_1^m = q - M_\phi p \left( 1 - b \ln \left( \frac{p}{p_c} \right) \right) r_1^m \quad f_2^m = p - d p_c r_2^m$$

$$M_\phi = M_\phi^c g(\theta)$$

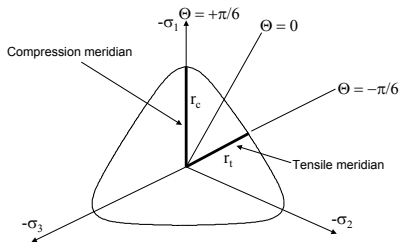
$$g(\theta) = \left( \frac{1 - \alpha \sin(3\theta)}{1 - \alpha} \right)^m \quad \sin(3\theta) = -\frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{\frac{3}{2}}} \quad m = -0.229$$

$$\alpha = \frac{k^{\frac{1}{n}} - 1}{k^{\frac{1}{n}} + 1} \leq 0.7925$$

# Yield surfaces (continuation)



(a) Section by  $p$ - $q$  plane



(b) Section by deviatoric plane

$$f_1^m = q - M_\phi p \left( 1 - b \ln \left( \frac{p}{p_c} \right) \right) r_1^m \quad f_2^m = p - d p_c r_2^m$$

- $r_1^m$ ,  $r_2^m$  are hardening parameters for deviatoric and volumetric mechanism
- $p_c$  is the critical pressure
- $M_\phi^c$  is the slope of critical state line measured along compression meridian

$$M_\phi^c = \frac{6 \sin(\phi)}{3 - \sin \phi}$$

# Plastic flow rule for deviatoric mechanism

- Plastic flow rule is assumed to be associated in the deviatoric plane while the corresponding volumetric part is replaced by an enhanced counterpart; the flow vector  $\mathbf{b}_1$  is defined as follows

$$\mathbf{b}_1^m = \text{dev} \left( \frac{\partial f_1^m}{\partial \boldsymbol{\sigma}} \right) - \frac{1}{3} \mathbf{1} \psi_v^m \quad \text{and} \quad \psi_v^m = M_\psi - \frac{q}{p}$$

- By analogy to the expression for  $M_\phi$   $M_\psi = M_\psi^c g(\theta)$  and

$$M_\psi^c = \frac{6 \sin(\psi)}{3 - \sin(\psi)} \quad \text{where } \psi \text{ is a dilatancy angle}$$

## Remarks

- Term  $\psi_v^m$  holds true for stress states remaining on the deviatoric yield surface  $f_1^m$
- By default  $\psi = \phi$

# Plastic flow rule for volumetric mechanism

- Plastic flow rule is assumed to be associated with the yield surface  $f_2^m$
- Plastic flow vector  $\mathbf{b}_2^m$  is defined as follows

$$\mathbf{b}_2^m = \frac{\partial f_2^m}{\partial \boldsymbol{\sigma}} = -\frac{1}{3}\mathbf{1}$$

# Evolution law for hardening parameter $r_1^m$

- In the original version of the model the evolution law for  $r_1^m$  parameter is defined as follows:

$$\dot{r}_1^m = \dot{\gamma}^p \frac{(1 - r_1^m)^2}{a_m} = \dot{\lambda}_1^m \frac{(1 - r_1^m)^2}{a_m}$$

For constant  $a_m$  material parameter integrating the above expression yields a hyperbolic dependence of  $r_1^m$  with respect to  $\gamma^p$

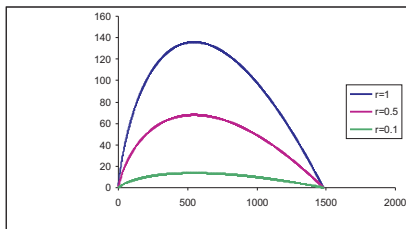
$$r_k^m = \frac{\gamma_k^p}{a_m + \gamma_k^p}$$

- Better matching of  $\frac{G}{G_o}(\gamma)$  curves (after Mellal) to the real measurements are obtained for modified law

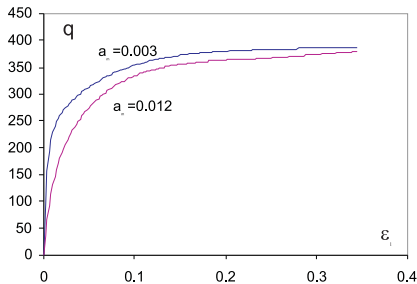
$$\dot{r}_1^m = \dot{\gamma}^p \frac{\left(1 - (r_1^m)^2\right)^2}{2 r_1^m a_m} = \dot{\lambda}_1^m \frac{\left(1 - (r_1^m)^2\right)^2}{2 r_1^m a_m}$$

# Evolution law for hardening parameter $r_1^m$

$r_1^m$  parameter reflects level of mobilization of the mechanism



Evolution of yield surface with  $r_1^m$



Influence of  $a_m$  parameter

## Evolution law for $r_2^m$ parameter

- The evolution law for the degree of mobilization of volumetric mechanism is defined as follows:

$$\dot{r}_2^m = \dot{\lambda}_2 \frac{(1 - r_2^m)^2}{c_m}$$

- This law yields a hyperbolic dependence of  $r_2^m$  with respect to  $\varepsilon_{v_2}^p$  (volumetric plastic strain generated exclusively by volumetric mechanism)
- $c_m$  is a material parameter

**NB. To simplify the model we assume  $r_2^m = 1$  (mechanism is fully mobilized)**

# Evolution law for critical pressure $p_c$

- The critical pressure  $p_c$  is a hardening parameter coupling both deviatoric and volumetric mechanism
- The evolution law for  $p_c$  parameter is defined as follows:

$$\dot{p}_c = -\beta p_c \dot{\varepsilon}_v^p$$

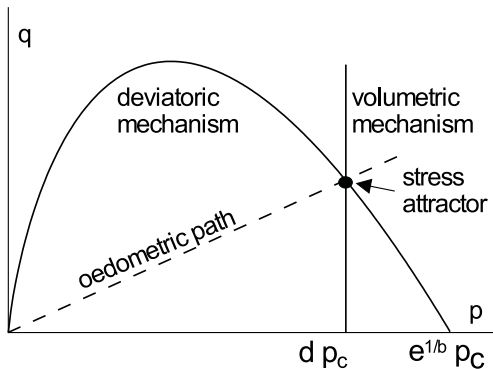
where  $\dot{\varepsilon}_v^p$  is the volumetric plastic strain rate accumulated from all active mechanisms

All material parameters are classified in the following five groups:

- Elastic parameters:  $G_a, \nu, n$
- Critical state parameters:  $\beta, \phi, \psi$
- Parameters for deviatoric hardening:  $a_m, b$
- Parameters for volumetric hardening:  $c_m, d$
- Limits of elastic behavior:  $r_1^{el}, r_2^{el}$

# Oedometer test

- In the oedometric test  $K_o$  ( $K_o = \frac{\sigma_h}{\sigma_v}$ ) depends exclusively on  $\phi$ ,  $d$  and  $b$  parameters, and it tends to a constant value with an increasing load
- Stress path is attracted by the intersection of the deviatoric and volumetric yield surfaces



# Oedometer test

## Assumptions:

- With an increasing load deviatoric and volumetric mechanisms become fully mobilized ( $r_1^m \rightarrow 1$  and  $r_2^m \rightarrow 1$ ) hence in the further derivations we will explicitly set them equal to 1
- As the stress state is attracted by the intersection point, the two yield conditions must be satisfied simultaneously

$$q - p M_\phi \left( 1 - b \ln \frac{p}{p_c} \right) = 0$$

$$p - d p_c = 0$$

Form the second equation one can evaluate  $\frac{p}{p_c} = d$  and then substitute it into the first equation which reads

$$q - p M_\phi (1 - b \ln(d)) = 0$$

# Oedometer test

## Derivation

- In the oedometric test  $\sigma_x = \sigma_z$   
 $q = (\sigma_x - \sigma_y)$   
 $p = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = -\frac{1}{3}(2\sigma_x + \sigma_y)$
- Expressing  $\sigma_x$  and  $\sigma_y$  through  $p$  and  $q$  one may find that  $K_o^{NC}$  (for state of normal consolidation), is expressed by the formula

$$K_o^{NC} = \frac{\sigma_x}{\sigma_y} = \frac{1 - \frac{1}{3}\frac{q}{p}}{1 + \frac{2}{3}\frac{q}{p}} = \frac{1 - \frac{1}{3}\xi}{1 + \frac{2}{3}\xi}$$

where

$$\xi = (1 - b \ln d) M_\phi^c = (1 - b \ln d) \frac{6 \sin(\phi)}{3 - \sin(\phi)}$$

# Oedometer test

## Derivation

- It is possible to get an exact match between  $K_o^{NC}$  and Jaky's  $K_o$

$$K_o^{NC} = \frac{1 - \frac{1}{3}\xi}{1 + \frac{2}{3}\xi} = 1 - \sin(\phi)$$

This way parameters  $b$  and  $d$  become dependent as follows

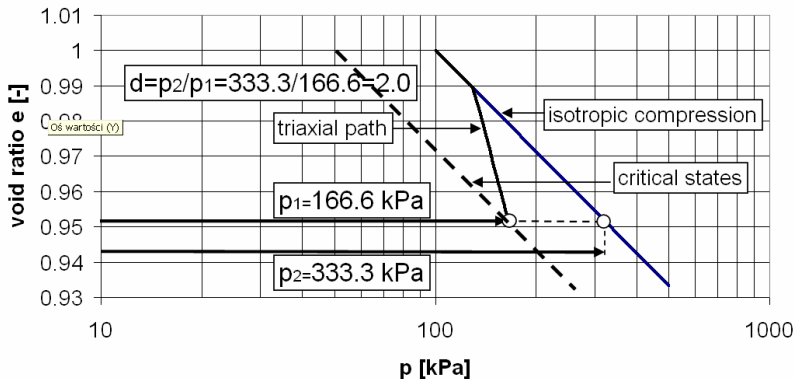
$$b = \frac{3}{2 \ln(d)} \frac{1 - \sin(\phi)}{(3 - 2 \sin(\phi))} \quad d = e^{\left( \frac{3}{2b} \frac{1 - \sin(\phi)}{3 - 2 \sin(\phi)} \right)}$$

- $d$  is usually equal  $d = 2$

## Remark

$K_o^{NC}$  value generated by the model is crucial for soil-structure interaction problems

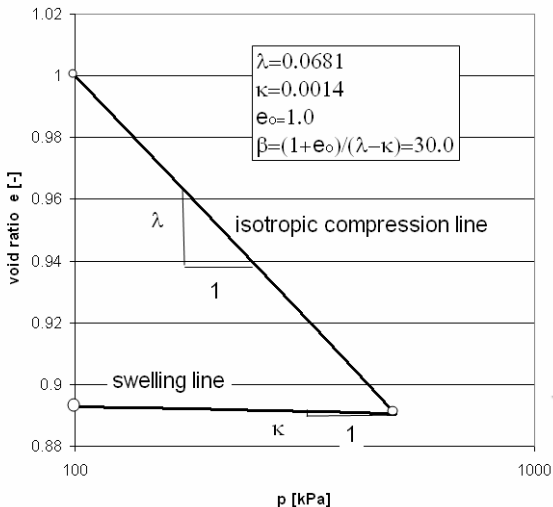
# Estimation of $d$ parameter



- Remark

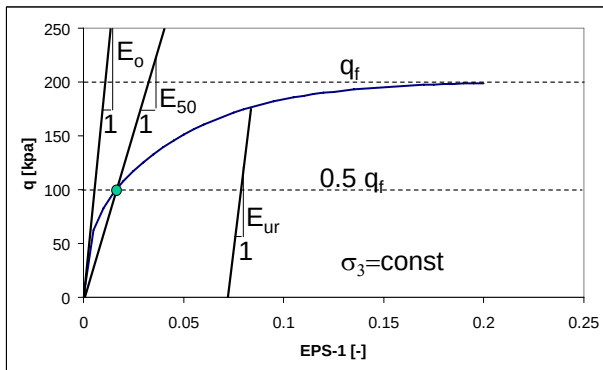
Assuming low values of  $b$  parameter and assuming  $d = 2$  (Hujeux 1985) the  $K_o^{NC}$  will be underestimated, hence  $b$  should be derived from the oedometric estimation for the assumed  $d$  value.

## Isotropic compression/oedometric test



# Notion of the initial/secant and unloading/reloading stiffness moduli

$$G_{ur} = \frac{E_{ur}}{2(1 + \nu_{ur})}$$



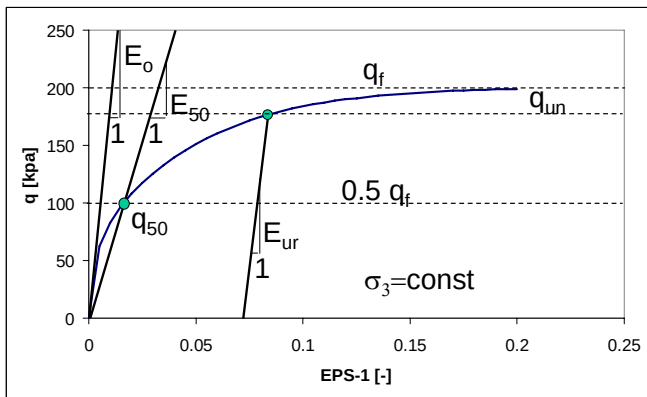
To simplify the model we assume the  $\nu$  parameter to be equal to  $\nu = \nu_{ur}$  and  $\nu_{ur}$  usually varies in the range  $\nu_{ur} = 0.2..0.3$ .

# $G_a$ & $n$ parameters in the range of small strains

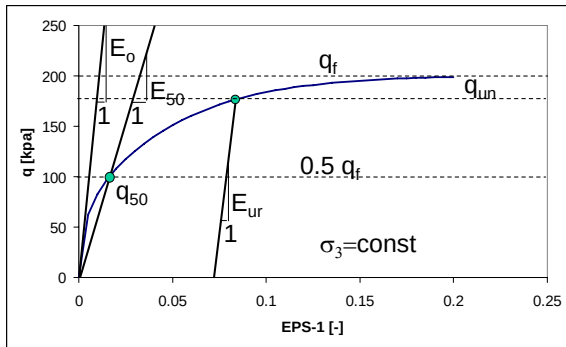
| Class of soil                                  | Void ratio $e_o$ | Index of plasticity   | $G_a$                                       | $n$  | Formula by..         |
|------------------------------------------------|------------------|-----------------------|---------------------------------------------|------|----------------------|
| Reconstituted clean sands                      | 0.64..0.81       | —                     | $(14300..16600) \frac{(2.17 - e)^2}{1 + e}$ | 0.4  | Iwasaki and Tatsuoka |
| Undisturbed sands without fines content        | 0.64..0.81       | —                     | $(7900..14300) \frac{(2.17 - e)^2}{1 + e}$  | 0.4  | Iwasaki and Tatsuoka |
| Undisturbed sands with fines content above 50% | 0.64..0.81       | —                     | $(2360..3090) \frac{(2.17 - e)^2}{1 + e}$   | 0.6  | Higuchi              |
| Clays                                          | 0.6..1.5         | low plasticity index  | $3270 \frac{(2.97 - e_o)^2}{1 + e_o}$       | 0.5  | Hardin and Black     |
| Clays                                          | 1.5..2.5         | high plasticity index | $445 \frac{(4.40 - e_o)^2}{1 + e_o}$        | 0.5  | Marcuson and Wahls   |
| Soft alluvial clays                            | 1.5..4.0         | 0.4..1.00             | $90 \frac{(7.32 - e_o)^2}{1 + e_o}$         | 0.6  | Kokusho              |
| Crushed rock                                   | 0.39..0.58       | —                     | $13000 \frac{(2.17 - e_o)^2}{1 + e_o}$      | 0.55 | Kokusho and Esashi   |
| Round gravel                                   | 0.32..0.43       | —                     | $8400 \frac{(2.17 - e_o)^2}{1 + e_o}$       | 0.60 | Kokusho and Esashi   |
| Railroad ballast                               | 0.68..0.78       | —                     | $7230 \frac{(2.97 - e_o)^2}{1 + e_o}$       | 0.38 | Prange               |

# Estimation of $G_a$ from drained triaxial test

- $G_a$  parameter should be equal to  $G_{ur}$
- To estimate  $G_{ur}$  we have to know  $n$  and  $\nu$
- The default value of  $\nu = \nu_{ur} = 0.2$  can be used while  $n$  can be taken from empirical formula (usually  $n = 0.5$ ).



# Estimation of $G_a$ from drained triaxial test



$$G_a \left( \frac{\sigma_3 + \frac{q_{un}}{3}}{p_{ref}} \right)^n = \frac{E_{ur}}{2 (1 + \nu_{ur})} \rightarrow G_a = \frac{\frac{E_{ur}}{2 (1 + \nu_{ur})}}{\left( \frac{\sigma_3 + \frac{q_{un}}{3}}{p_{ref}} \right)^n}$$

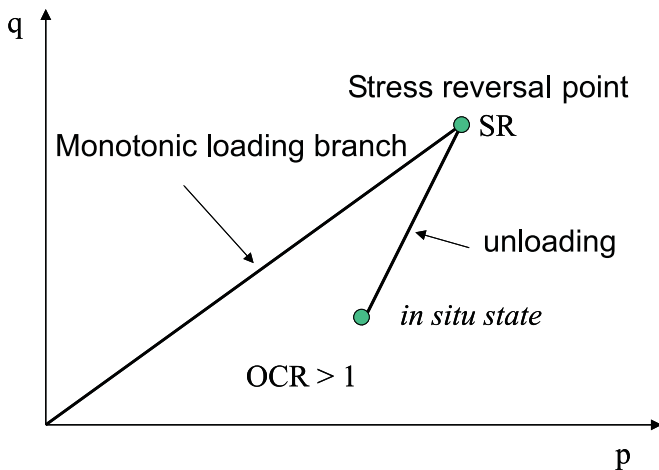
NB. If we have two values of  $E_{ur}$  for two  $\sigma_3$  values we may estimate also  $n$  parameter

# Estimation of $a_m$ parameter

- Once all parameters of the model, except  $a_m$ , are fixed one may simulate a single-element drained strain controlled triaxial test under constant confining stress  $\sigma_3 = \text{const}$  for few assumed  $a_m$  values to fit demanded ratio  $\frac{E_{50}}{E_{ur}}$ .
- In case when ratio  $\frac{E_{50}}{E_{ur}}$  is not known from the experiment the default value can be taken as  $\frac{E_{50}}{E_{ur}} = 0.33$
- Larger values of  $a_m$  parameter yield softer response in  $q - \varepsilon_1$  curves

# State parameters

We need to estimate  $p_c$  and  $r_1$  parameters with aid of  $K_o^{SR}$  and  $OCR$  plus given  $\sigma_o$  state



# Simulation of oedometer test

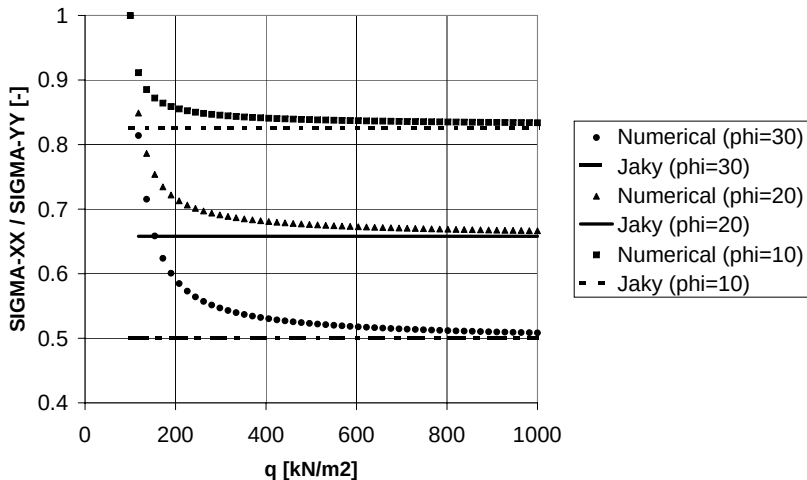


Figure: Variation of  $\frac{\sigma_x}{\sigma_y}$  ratio during oedometric compression

# Simulation of triaxial drained compression test (OCR=1)

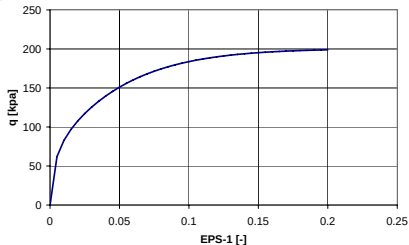


Figure: Shear characteristic in the triaxial compression test (OCR=1)

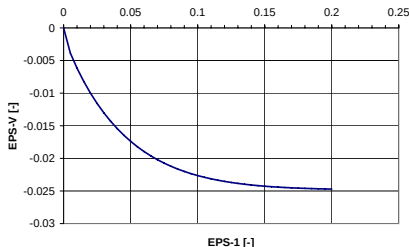


Figure: Dilatancy characteristic in the triaxial compression test (OCR=1)

# Simulation of triaxial drained compression test (OCR=5)

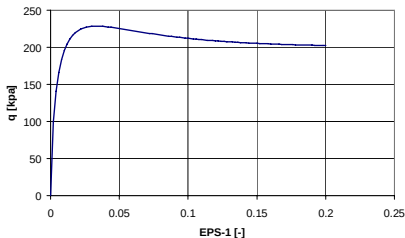


Figure: Shear characteristic in the triaxial compression test (OCR=5)

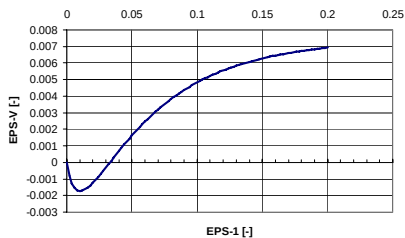


Figure: Dilatancy characteristic in the triaxial compression test (OCR=5)