Enhanced modeling of creep and aging in damage plasticity model for concrete

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Enhanced aging and creep EC2 compatible module coupled with damage plasticity model

1. Motivation: the **previous approach** works fine when load application time instances are not far from each other (ie. t_1 =7 [d] then t_2=120 [d])
2. In the EC2 (EN 1992-1-1:2004+AC:2008) **creep** is assumed as **viscoelastic**
3. **Formula** $\beta_{cc}(t)$ in EC2 **is valid for** $t_o \geq 3$ **[d]**
4. **Creep/aging** module is supported in **continuum** and **shell elements**
5. Note that **strength and stiffness** do not develop with the same rate during maturing
6. **Creep in tension** is **linear** while in **compression** it is **nonlinear**

Aging and creep according to the EC2 standard

- Time dependent creep coefficient: $\phi(t, t_o) = \phi_o \beta_c(t, t_o)$
- **Basic creep coefficient**: $\phi_o = \phi_{RH} \beta(f_{cm}) \beta(t_o)$
- $\phi_{RH} = \begin{cases} 1 + \dfrac{1 - \dfrac{RH}{100}}{0.1\ h_o^{1/3}} & \text{for} \quad f_{cm} \leq 35 \text{ MPa} \\ \left(1 + \dfrac{1 - \dfrac{RH}{100}}{0.1\ h_o^{1/3}} \alpha_1 \right) \alpha_2 & \text{for} \quad f_{cm} > 35 \text{ MPa} \end{cases}$
- $\beta(f_{cm}) = \dfrac{16.8}{\sqrt{f_{cm}}}$ (**effect of class of concrete**)
- $\beta(t_o) = \dfrac{1}{0.1 + t_o^{0.2}}$ (**effect of load application time**)
- $\beta_c(t, t_o) = \left[\dfrac{t - t_o}{\beta_H + t - t_o} \right]^{0.3}$ → **creep evolution**

Temperature adjusted time

- Time parameter t and t_o can be replaced by a corresponding temperature adjusted value t_T defined as follows
- $t_T(t) = \int_{t_1}^{t} \exp(-\frac{Q}{R}\left(\frac{1}{273+T(\tau)} - \frac{1}{273+T_{ref}}\right) d\tau$
- $\frac{Q}{R} = 4000 \text{ K}, \; T_{ref} = 20 \; [^oC]$

Implementation in ZSoil 2016

- Implementation scheme was partially based on the algorithm given in PhD by Havlásek
- Creep strain increment is computed using the following scheme (Kelvin chain of units)

$$\Delta \varepsilon^{cr}_{n+1} = \mathbf{D}_o^{-1} \frac{1}{v^{cr}_{n+1/2}} \sum_{\mu=1}^{N} A_\mu \left(1 - \beta_{\mu,n+1}\right) \boldsymbol{\sigma}_{v\mu,n}$$

where:

  - $\mathbf{D}_o^{-1}$ is an elastic compliance matrix computed for unit Young's modulus
  - $v^{cr}_{n+1/2}$ is an extra scaling factor amplifying creep rate due to aging phenomenon (here it is not equivalent to the fraction of solidified layers)
  - $\boldsymbol{\sigma}_{v\mu,n+1}$ represents viscous effective stresses in $\mu - th$ Kelvin unit
  - A_μ is the ultimate creep strain value in μ-th Kelvin unit

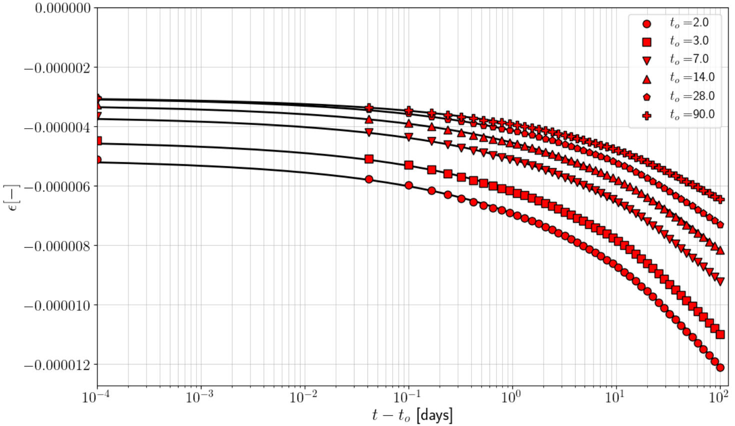
Implementation in ZSoil 2016

- Viscous stress update:
  $\boldsymbol{\sigma}_{v\mu,n+1} = \beta_{\mu,n+1}\boldsymbol{\sigma}_{v\mu,n} + \lambda_{\mu,n+1}\Delta\overline{\boldsymbol{\sigma}}_{n+1}$
- $\lambda_{\mu,n+1} = (1 - \beta_{\mu,n+1})\dfrac{\tau_\mu}{\Delta t_{n+1}}$
- The algorithmic effective Young's modulus is expressed as follows
  $$\overline{E} = \frac{1}{\dfrac{1}{v^E E_{28}} + \dfrac{1}{v^{cr}} \sum_{\mu=1}^{N} (1 - \lambda_{\mu,n+1}) A_\mu}$$
- $v^E = \sqrt{\beta_{cc}}$
- $\beta_{cc} = \begin{cases} \exp(s(1 - \sqrt{28/t})) & \text{for} \quad t \leq 28 \text{ days} \\ 1 & \text{for} \quad t > 28 \text{ days} \end{cases}$

Derivation of v^{cr} function (ZSoil 2016)

- Evolution of creep strain in time, according to EC2, can be expressed by the following equation
  $$\varepsilon^{cr} = A_1 \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o)$$
  where $A_1 = \phi_{RH} \beta(f_{cm}) / E_{28}$
- Evolution of the reference creep strain for concrete loaded at $t_o = 28$ days (matured concrete) can be defined as
  $$\varepsilon^{cr}_{ref} = A_1 \left(\frac{t - t_o}{\beta_H + t - t_o} \right)^{0.3} \beta(t_o = 28)$$
- The reference creep strain curve is taken here as a basis for optimization of A_μ coefficients in chain of nonaging Kelvin units (retardation times τ_μ are predefined by considering duration of carried out analysis time)
- To derive v^{cr} we assume the following creep strain rates compatibility condition
  $\dot{\varepsilon}^{cr} = \frac{1}{v^{cr}} \dot{\varepsilon}^{cr}_{ref}$ (**here is the source of limitation**)
- This yields the following definition of v^{cr} function
  $$v^{cr} = \frac{\beta(t_o = 28)}{\beta(t_o)}$$
  where t_o is the age of concrete at the beginning of analysis

Benchmark: monotonic creep (continuum) (2016)



Damage plasticity

- In isotropic damage mapping tensor: $\mathbf{D} = \dfrac{1}{1-D}\mathbf{I}$
- Damage variable $0 \leq D \leq 1$
- Hence: $\boldsymbol{\sigma} = (1-D)\,\bar{\boldsymbol{\sigma}}$
- $\boldsymbol{\sigma} = (1-D)\,\mathbf{E} : (\varepsilon - \varepsilon^p - \varepsilon^c - \varepsilon^o)$
- Plastic flow rule: $\dot{\varepsilon^p} = \dot{\lambda}\dfrac{\partial G}{\partial \bar{\boldsymbol{\sigma}}}$
- In the reference model $D = 1 - (1 - D_c)(1 - s\ D_t)$
- ε^p, ε^c depend on **effective stresses** $\bar{\boldsymbol{\sigma}}$ (!)

New general approach

- Pure creep measure for the unit stress

$$C(t,\tau) = \underbrace{\frac{\beta(t_o = \tau)}{\beta(t_o = 28)}}_{\tilde{\beta}} \sum_i^N A_i \left(1 - e^{-\gamma(t-\tau)}\right)$$

- Creep function

$$\delta(t,\tau) = \frac{1}{E(\tau)} + \tilde{\beta} \sum_i^N A_i \left(1 - e^{-\gamma(t-\tau)}\right)$$

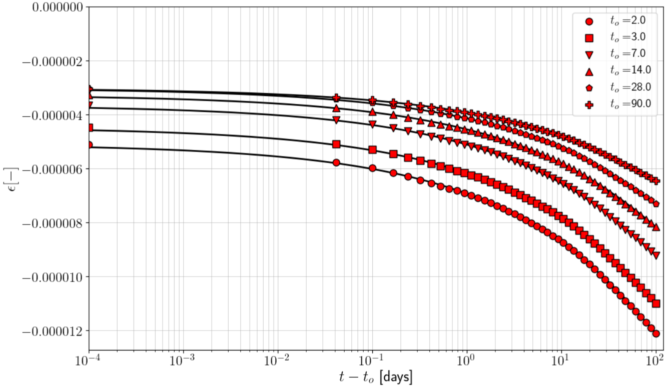
- In viscoelasticity

$$\sigma(t) =$$

$$E(t)\left\{\varepsilon(t) - \varepsilon_o(t) - \int_{t_o}^{t} - \left(\frac{\partial}{\partial\tau}\left(\frac{1}{E(\tau)}\right) + \frac{\partial C(t,\tau)}{\partial\tau}\right)\sigma(\tau)\mathrm{d}\tau\right\}$$

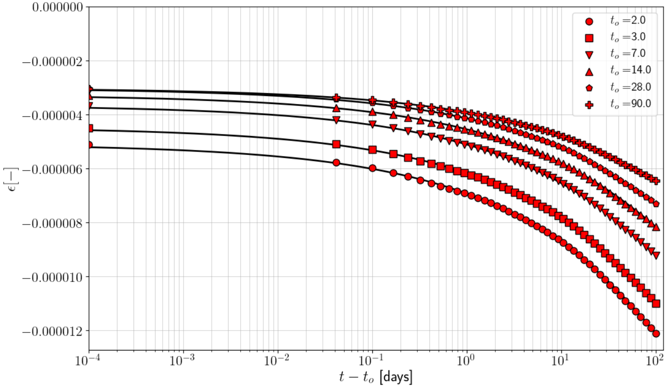
- $E(\tau) = \sqrt{\beta_{cc}(\tau)}E_{28}$

Monotonic loading: $t_o = 7$ [d] (v2016)



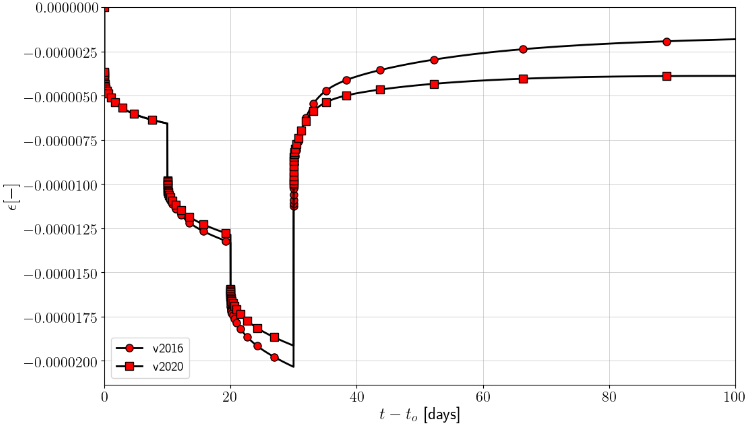
Version 2016

Monotonic loading: $t_o = 7$ [d] (v2020)



Version 2020

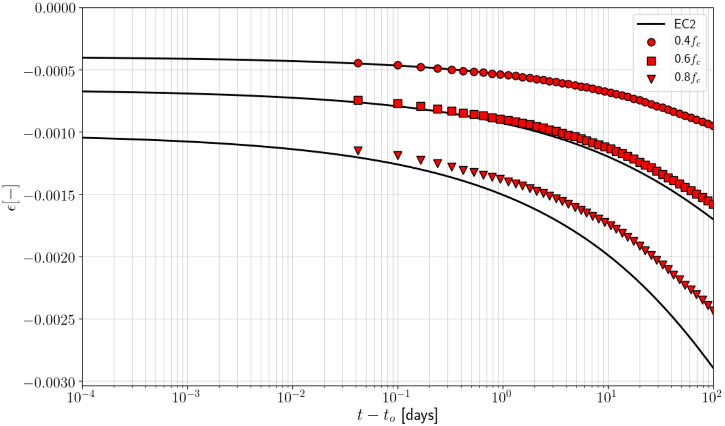
Variable loading: $t_o = 7$ [d] (v2016 vs v2020)



Source of creep nonlinearity (nominal vs effective stresses)

- In **tension**: $\sigma = \overline{\sigma}$ for $\sigma < f_t$ $(D = 0)$
- In **compression**: $\sigma = \begin{cases} \overline{\sigma} & \text{for} \quad \sigma \leq f_{co} \quad (D = 0) \\ \dfrac{\overline{\sigma}}{1 - D} & \text{for} \quad \sigma > f_{co} \quad (D > 0) \end{cases}$
- **At peak in compression** $D = \tilde{D}_c$ hence **creep strains will be amplified** by the factor $\dfrac{1}{1 - \tilde{D}_c}$
- The **two parameters** in the model: f_{co}, $\tilde{D}_c$ can **control creep amplification** in compression
- Amplification factor in EC2: $\phi = \phi^* \, e^{1.5(k_\sigma - 0.45)}$
- $k_\sigma = \begin{cases} 0.45 & \text{for} \quad \sigma \leq f_{co} \\ \dfrac{\sigma}{f_c} & \text{for} \quad \sigma > f_{co} \end{cases}$

Nonlinear creep in compression



Conclusions

- The **new enhanced creep+aging module** is implemented in the implicit format (time steps can be large)
- **For complex loading cases at stage of concrete maturing** the new creep module gives more accurate results than the old version
- **Amplification of creep** appears only **in compression** (effective compressive stress becomes higher than the nominal one when $\sigma > f_{co}$)
- **Amplification of tensile creep is not present** (effective tensile stress becomes constant in the post-peak branch)
- The old module will be kept till ZSoil 2020 and then it will be canceled