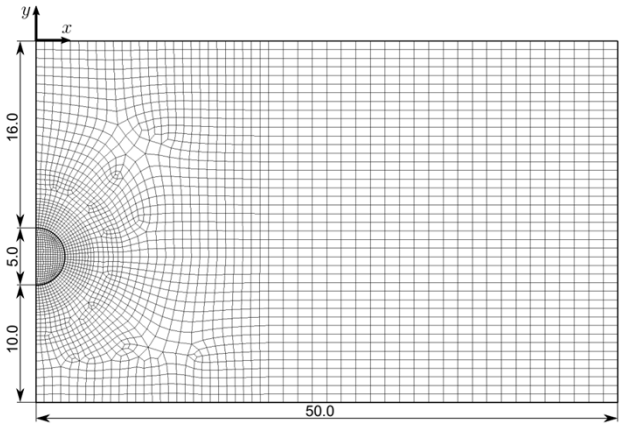
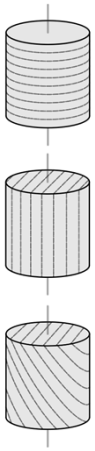
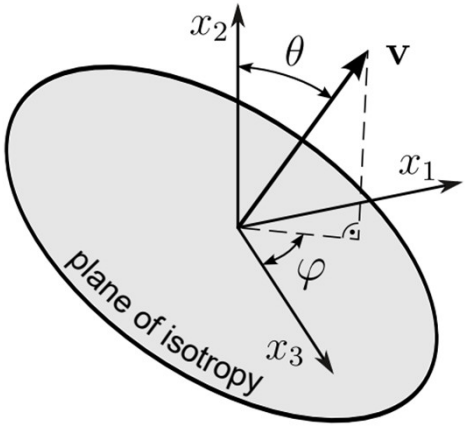
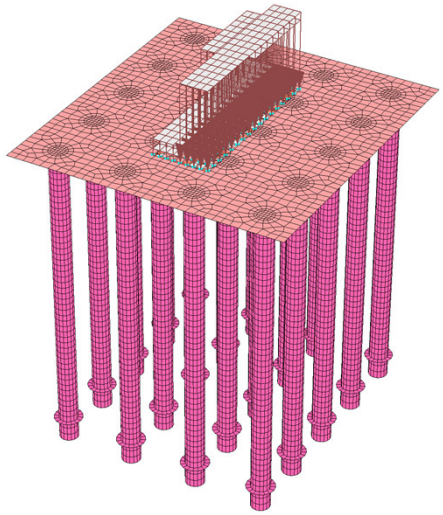
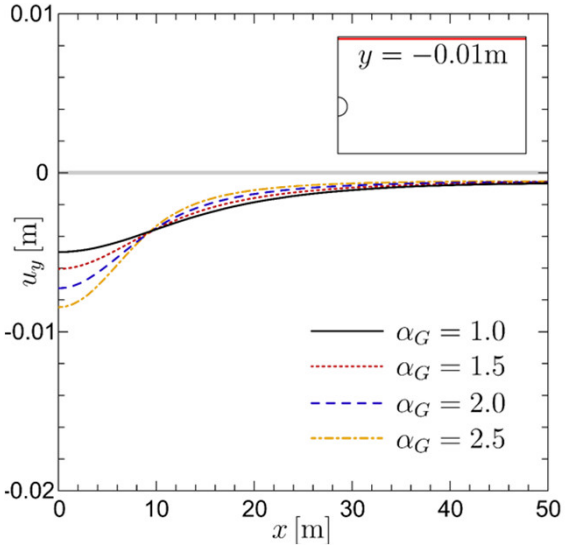
# Some experiences with the new ZSoil implementation of the AHS-Brick model in FE modelling








**Marcin Cudny, Katarzyna Lisewska**
Gdańsk University of Technology

**Artur Zachodni**
Keller Polska

**Andrzej Truty**
Cracow University of Technology

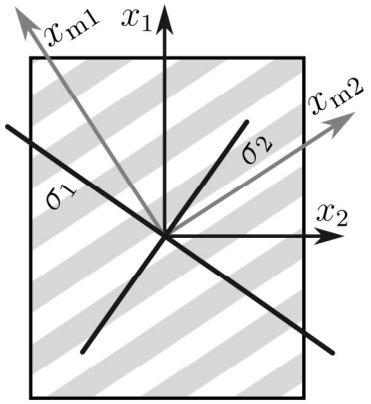
POLITECHNIKA GDAŃSKA

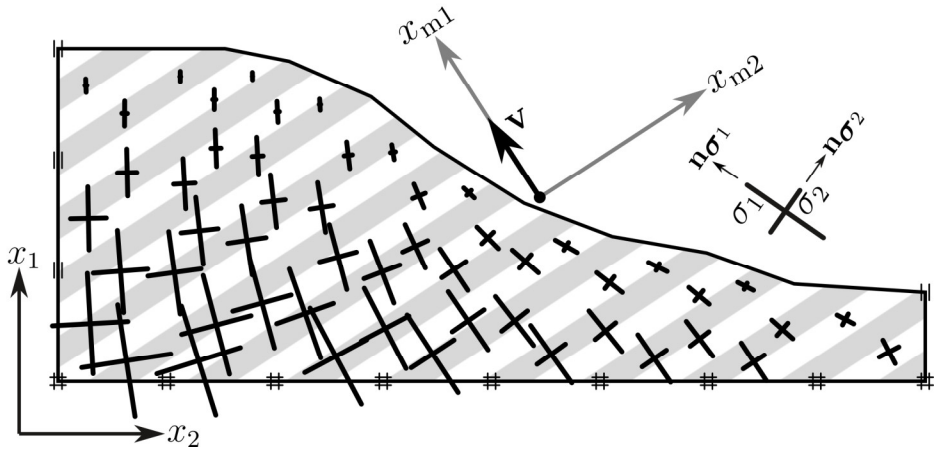
KELLER

Cracow University of Technology

**ZSoil Days, 29.08.2025**

# Anisotropy of mechanical properties (stiffness, strength), principal axes



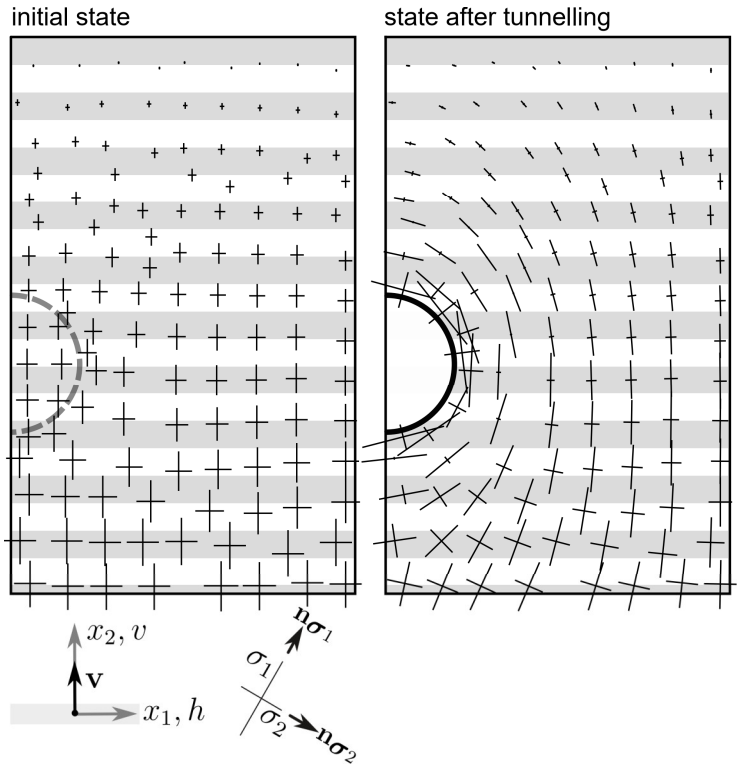

$x_i$ – geometrical axes (e.g. gravity)

$h, v$ – material axes ($x_m$,$y_m$), here cross-anisotropic microstructure

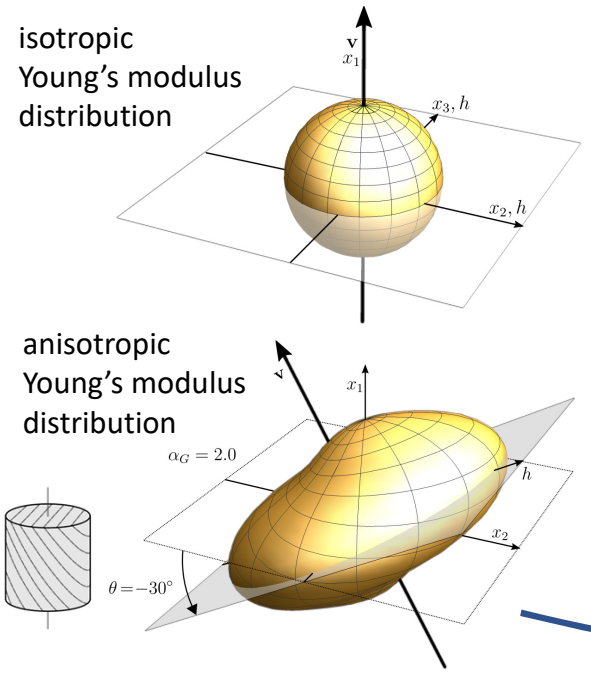
v – axis of symmetry (cross-anisotropy)

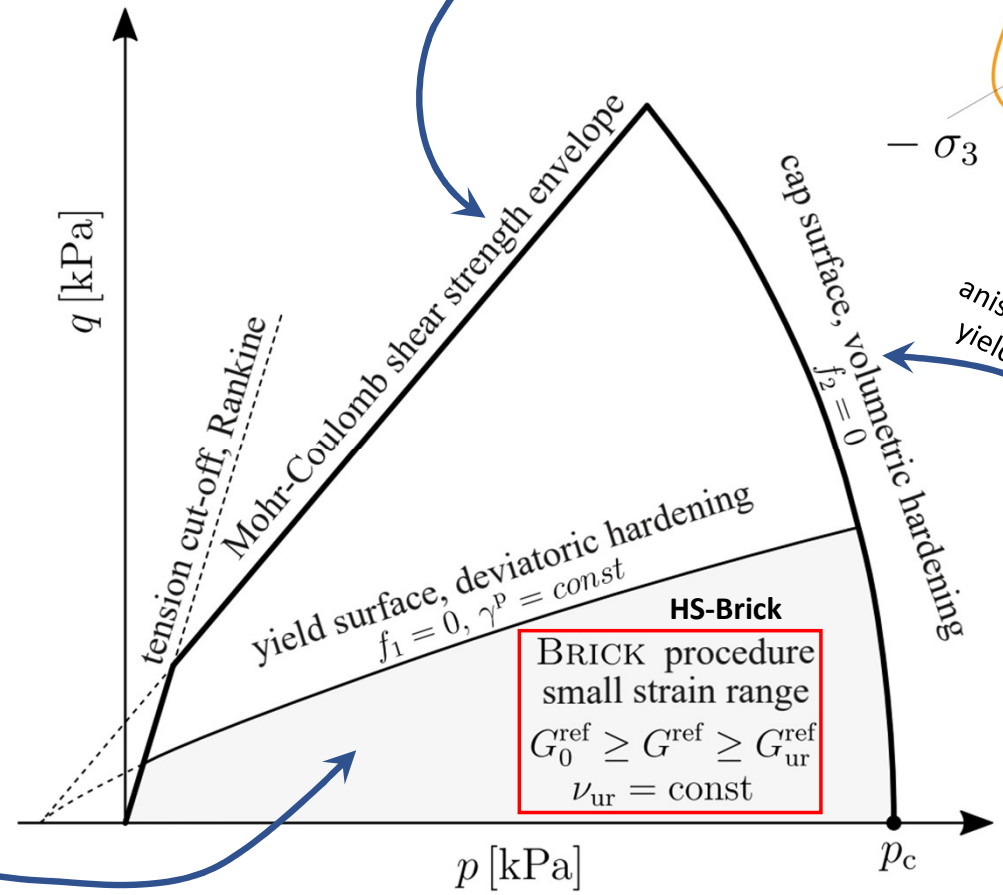
$\sigma_i$ – principal stresses

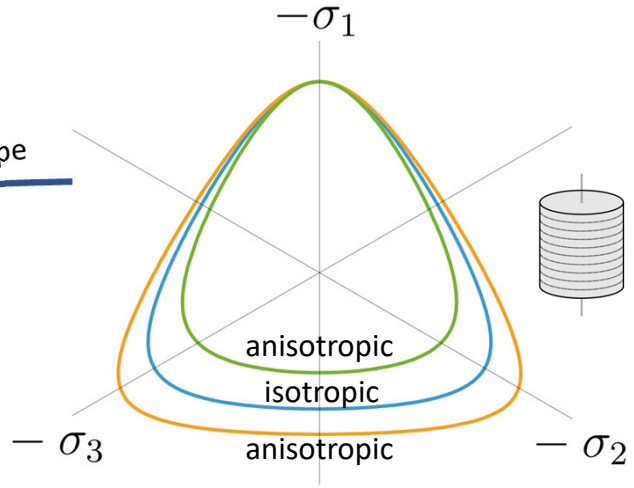
$n\sigma_i$ – principal stress directions



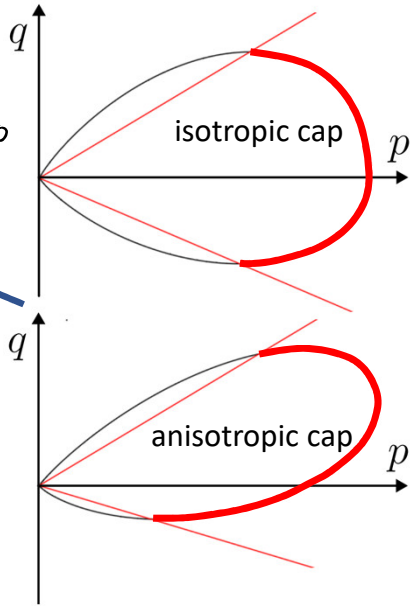
# HS-Brick model, possibilities to incorporate anisotropy



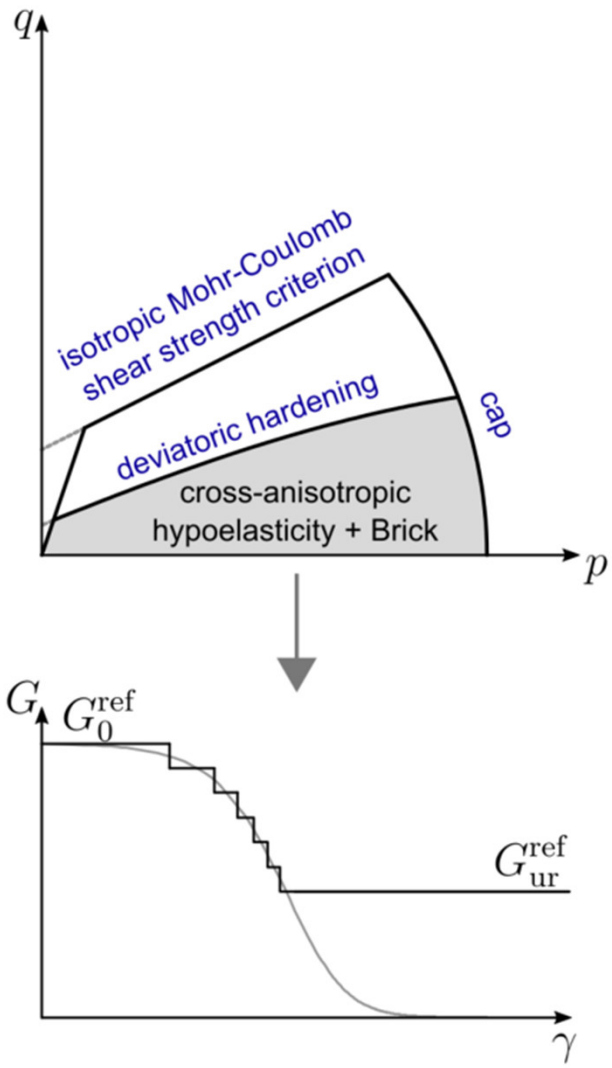




anisotropic cap yield surface



# AHS-Brick model assumptions

- Anisotropy is incorporated **only in the small-strain range** of the AHS-Brick model and extends to larger strains through the Brick procedure and hardening.
- The original hypoelastic stiffness (power law) is converted into cross-anisotropy through a special scaling procedure.
- Remaining elements of the model (shear strength criterion and two yield surfaces) are isotropic.
- A special numerical procedure is applied in the return mapping algorithm to maintain its robustness, originally developed under the assumption of isotropy (i.e., return in principal stress space).

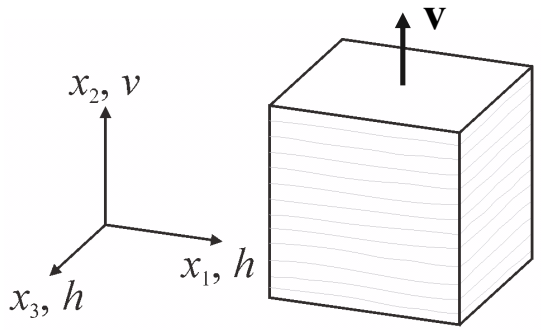



$$\gamma = \sqrt{\frac{3}{2} e_{ij} e_{ij}}$$

# Cross-anisotropic stiffness

Originally requires **5** independent parameters, e.g.:

$$E_v,\ E_h,\ G_{vh},\ G_{hh},\ \nu_{vh}$$




Graham & Houlsby (1983) proposed the following reduction to **3** independent parameters:

$$E^* = E_v,\ \nu^* = \nu_{hh},\ \alpha = \sqrt{\frac{E_h}{E_v}} = \frac{G_{hh}}{G_{vh}} = \frac{\nu_{hh}}{\nu_{vh}}$$

Mašín & Rott (2014) introduced distinct cross-anisotropic coefficients for Young's moduli, shear moduli, and Poisson's ratios:

$$\alpha_G = \frac{G_{hh}}{G_{vh}},\ \alpha_E = \frac{E_h}{E_v},\ \alpha_\nu = \frac{\nu_{hh}}{\nu_{vh}}$$

They also proposed cross-anisotropic exponents to establish relationships among the cross-anisotropic coefficients:

$$\alpha_G = \alpha_E^{x_{GE}},\ \alpha_G = \alpha_\nu^{x_{G\nu}}$$

| | |
|---|---|
| $x_{GE} = 0.5, x_{G\nu} = 1.0$ | Graham & Houlsby (1983) |
| $x_{GE} \approx 0.8, x_{G\nu} = ?$ | experimental evidence for stiff clays, Mašín & Rott (2014) |

# Cross-anisotropic hypoelastic stiffness via XA2 scaling,

Niemunis & Staszewska (2021)

Isotropic hypoelastic stifness (HS and HS-Brick models):

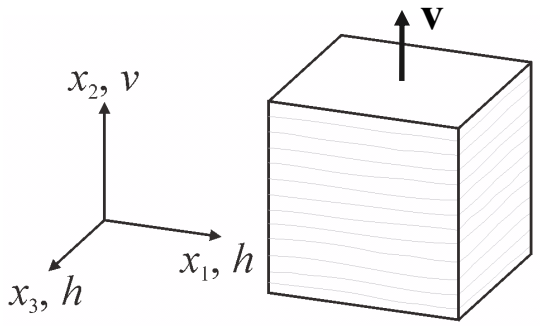
$$D^{\text{iso}}_{ijkl}(\boldsymbol{\sigma}) = \frac{E(\boldsymbol{\sigma})}{(1+\nu)(1-2\nu)}\left[\nu\delta_{ij}\delta_{kl} + \frac{1-2\nu}{2}(\delta_{ik}\delta_{jl}+\delta_{jk}\delta_{il})\right], \quad E(\boldsymbol{\sigma}) = E^{\text{ref}}\left(\frac{p}{p_{\text{ref}}}\right)^m$$

Isotropic stiffness into cross-anisotropic stiffnes via XA2 scaling:

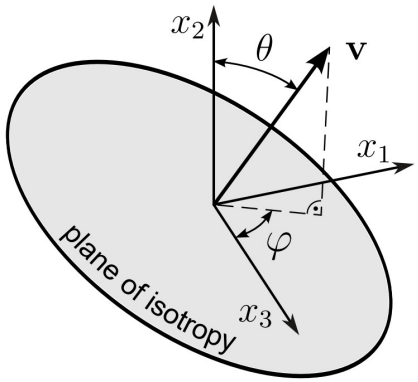
$$D^{\times A_2}_{ijkl} = Q_{abij} D^{\text{iso}}_{abcd} Q_{cdkl}, \qquad Q_{ijkl} = \mu_{ik}\mu_{jl} + cI_{ijkl}, \quad \mu_{ij} = a\delta_{ij} + bM_{ij} \;\Rightarrow\; \mathbf{Q}(\alpha_G, \beta, \mathbf{M})$$

Microstructure tensor **M** and symmetry axis **v**:

$$\mathbf{M} = \mathbf{v}\otimes\mathbf{v}, \quad M_{ij} = v_i v_j$$




$$\mathbf{v} = [0,1,0]^T$$
$$\mathbf{M} = \text{diag}(0,1,0)$$




$$\mathbf{v} = [\sin\varphi\sin\theta, \cos\theta, \cos\varphi\sin\theta]^T$$

The ability to control the coupling between $\alpha_G$ and $\alpha_E$ or $\alpha_G$ and $\alpha_\nu$:

$$\alpha_G = \alpha_E^{\beta/2} = \alpha_\nu^{\beta}$$

$$x_{GE} = \beta/2, \quad x_{G\nu} = \beta$$

# Cross-anisotropic parameters in AHS-Brick model

$$G_{vh}^{\mathrm{ref}} = E^{\mathrm{ref}} \frac{\alpha_G^{\frac{2}{\beta}-1}}{2(1+\nu)}$$

$$G_{hh}^{\mathrm{ref}} = E^{\mathrm{ref}} \frac{\alpha_G^{\frac{2}{\beta}}}{2(1+\nu)}$$

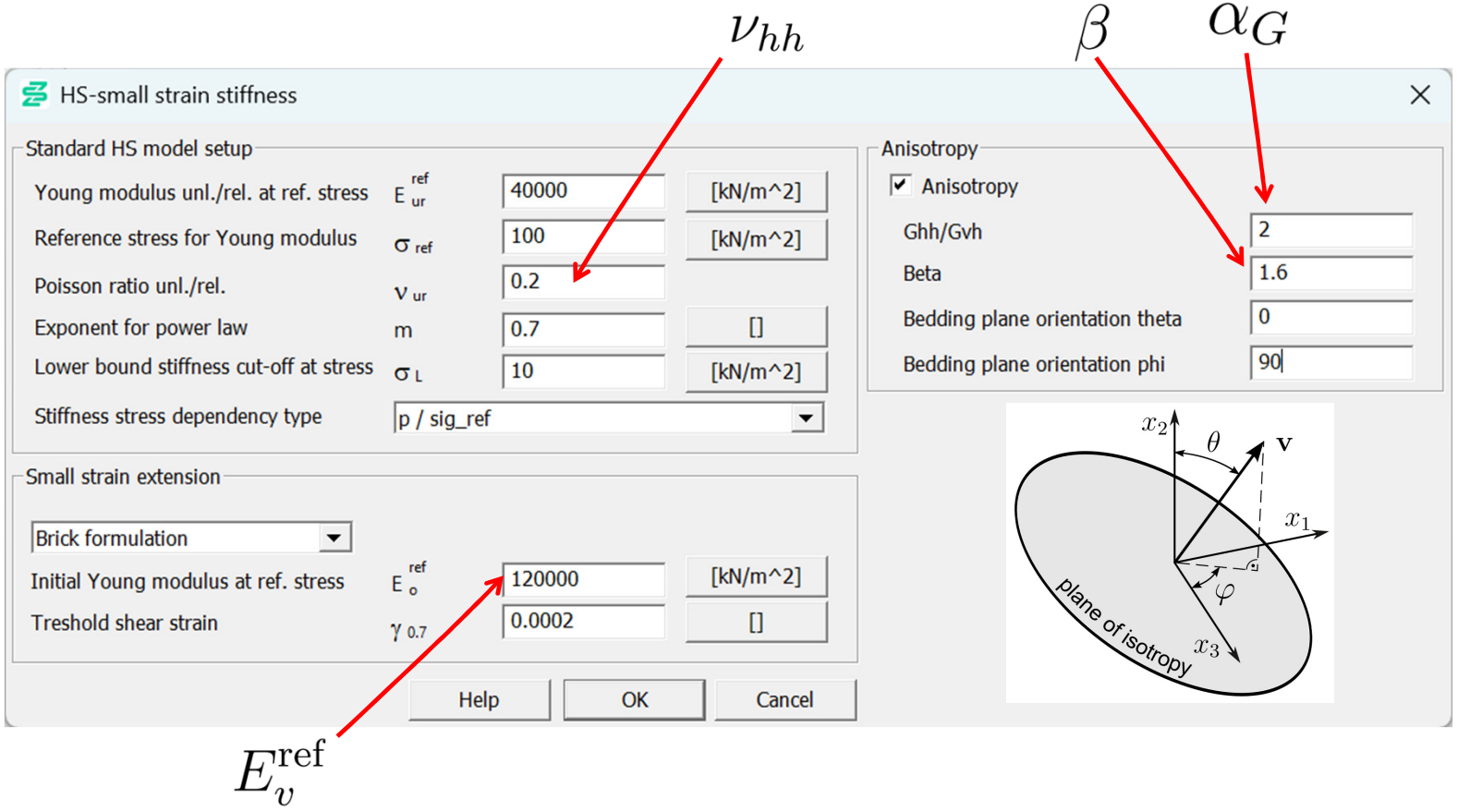
$$\boxed{E_v^{\mathrm{ref}} = E^{\mathrm{ref}}}$$

$$E_h^{\mathrm{ref}} = E^{\mathrm{ref}} \alpha_G^{\frac{2}{\beta}}$$

$$\nu_{vh} = \nu \alpha_G^{-\frac{1}{\beta}}$$

$$\boxed{\nu_{hh} = \nu}$$

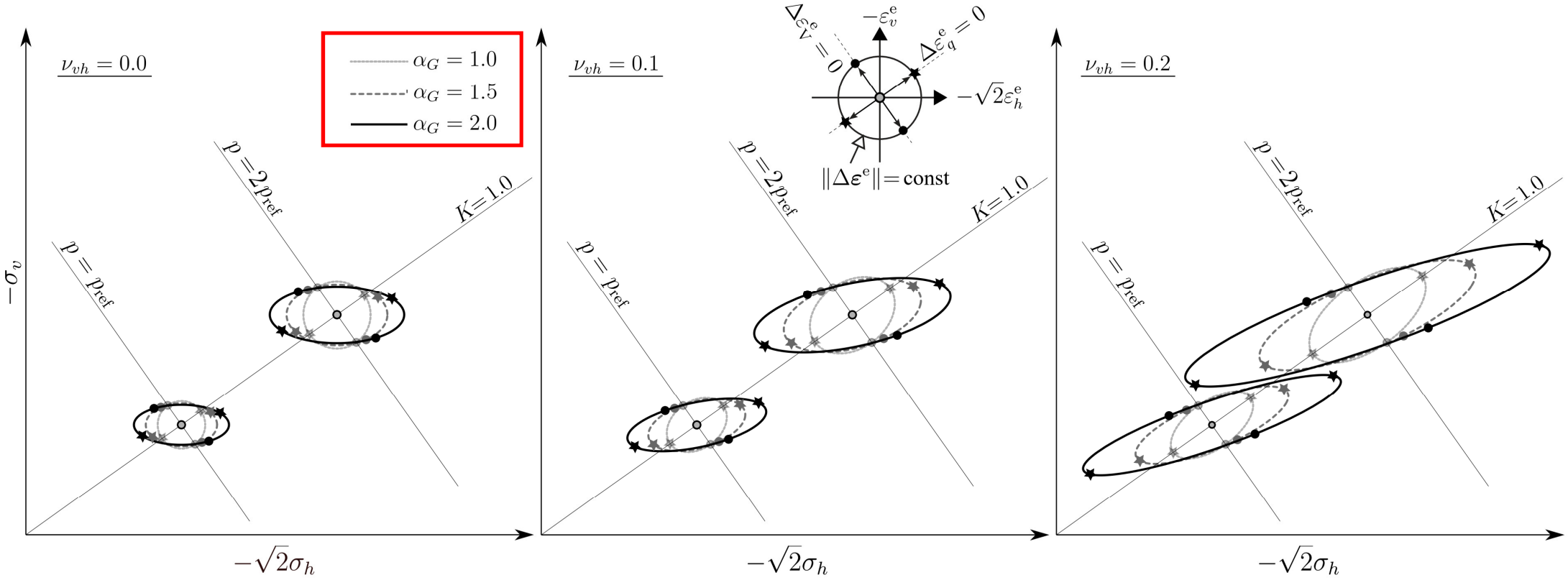
$$\nu_{hv} = \nu \alpha_G^{\frac{1}{\beta}}$$




set 1: $E^{\mathrm{ref}}, \nu, \alpha_G, \beta \rightarrow E_v^{\mathrm{ref}}, \nu_{hh}, \alpha_G, \beta$

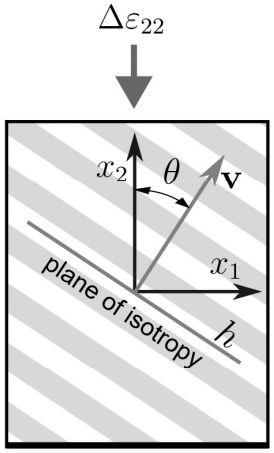
set 2: $G_{vh}^{\mathrm{ref}}, \nu_{vh}, \alpha_G, \beta$

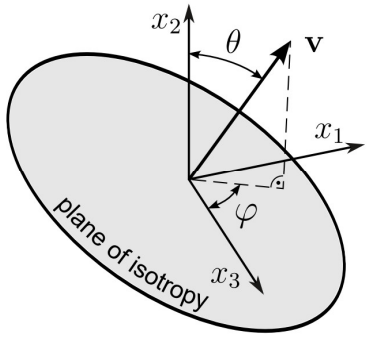
# Stress response envelopes for cross-anisotropic hypoelastic stiffness



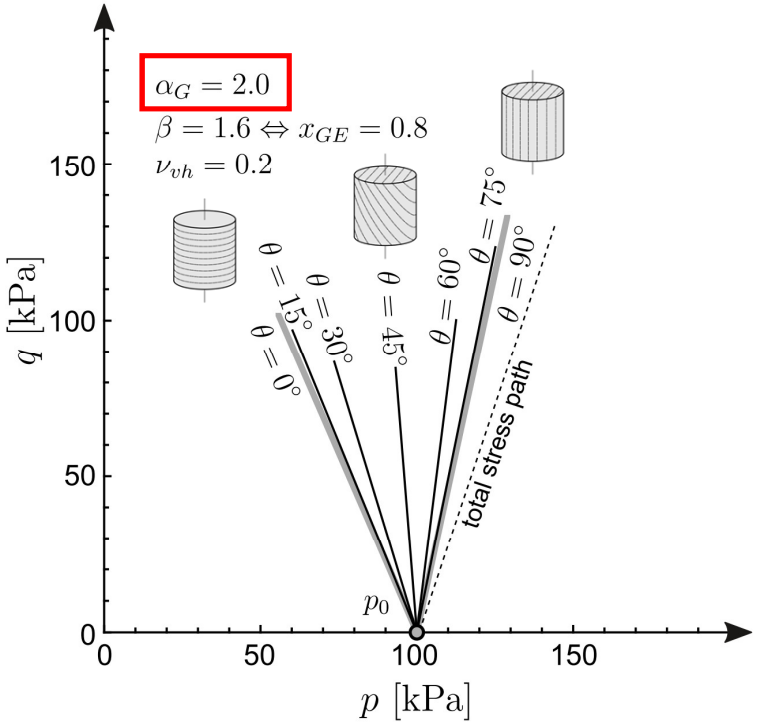

set 2: $G_{vh}^{\mathrm{ref}} = \mathrm{const},\ \nu_{vh},\ \alpha_G,\ \beta = 1.6$

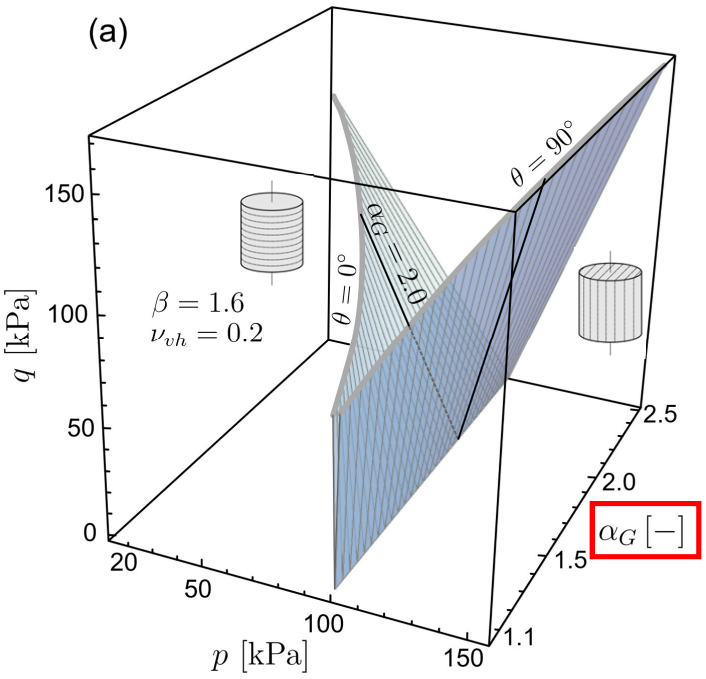
# The influence of cross-anisotropy coefficient $\alpha_G$ and the inclination angle of the plane of isotropy $\theta$ on the initial inclination of stress paths for TX CIU test



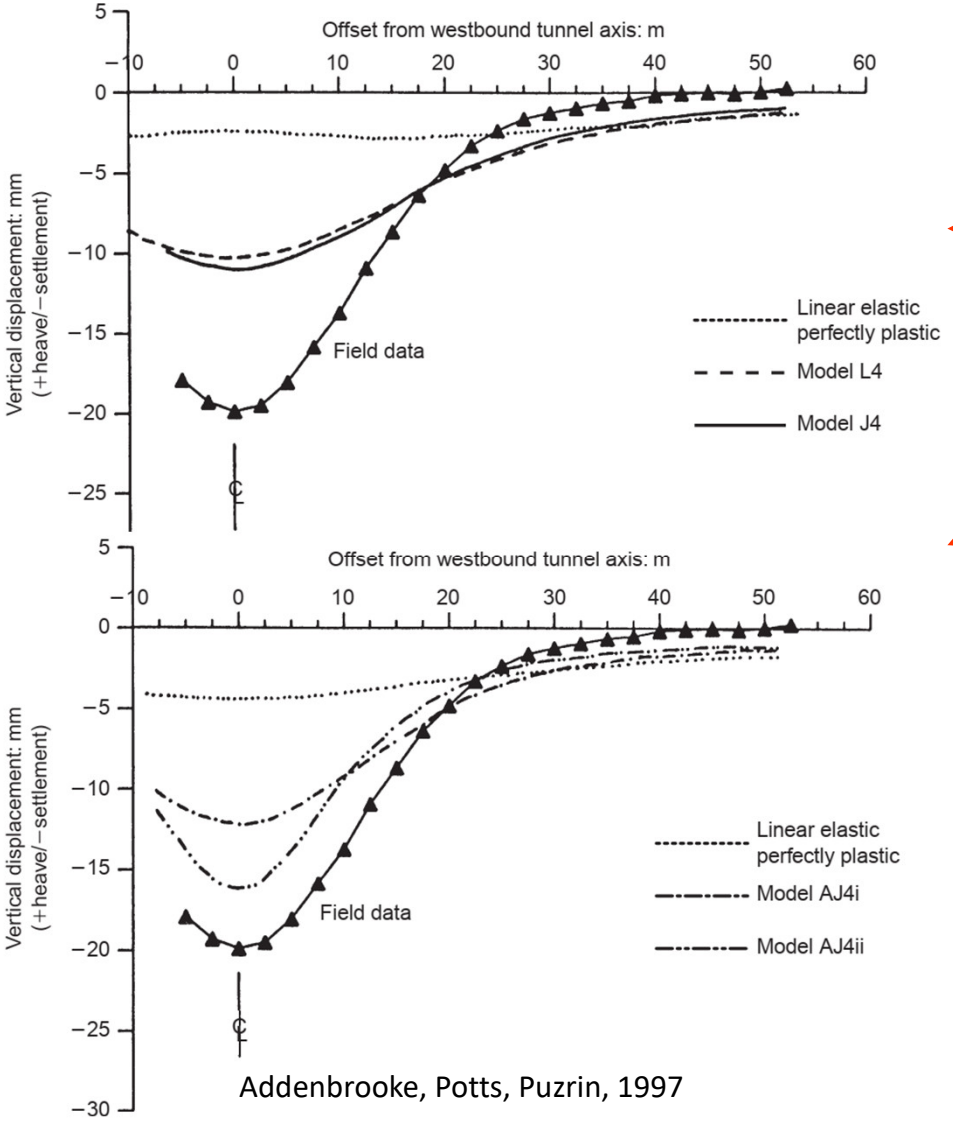




$\Delta\varepsilon_{22} = -5 \cdot 10^{-4}$
$\Delta\varepsilon_{\mathrm{V}} = 0$
$G_{vh}^{\mathrm{ref}} = 50000\,\mathrm{kPa}$
$p_0 = p_{\mathrm{ref}} = 100\,\mathrm{kPa}$
$\varphi = 90°$



# Experiences from numerical simulations of tunnelling induced deformation with different constitutive models

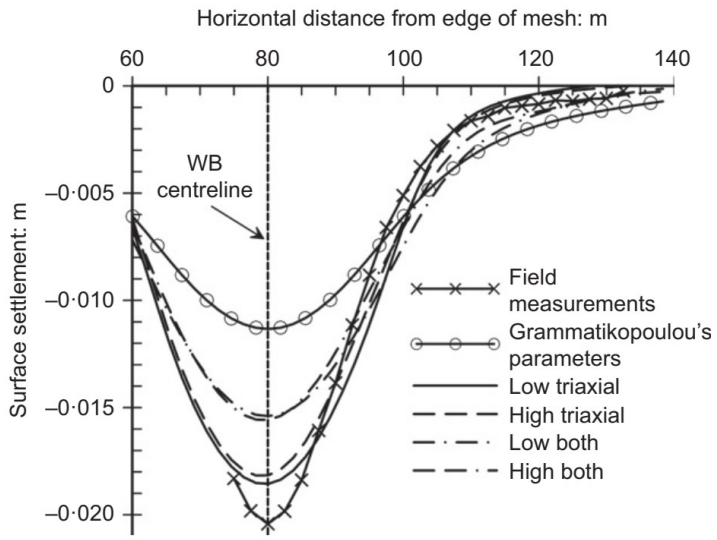



Addenbrooke, Potts, Puzrin, 1997

**Short term settlement trough, London Clay, open shield**

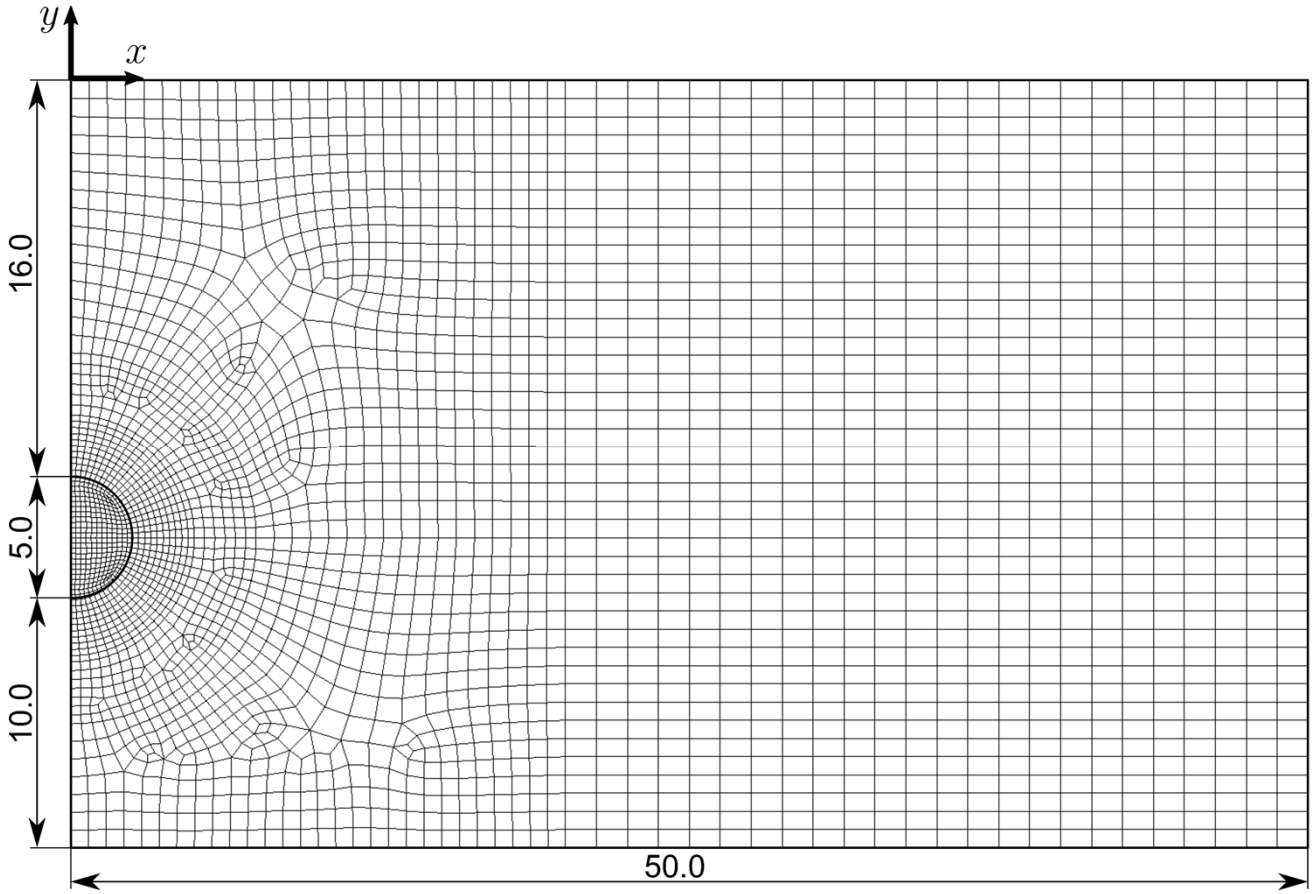
**isotropic models**
Addenbrooke, Potts, Puzrin, 1997

**anisotropic models (stiffness and permeability)**



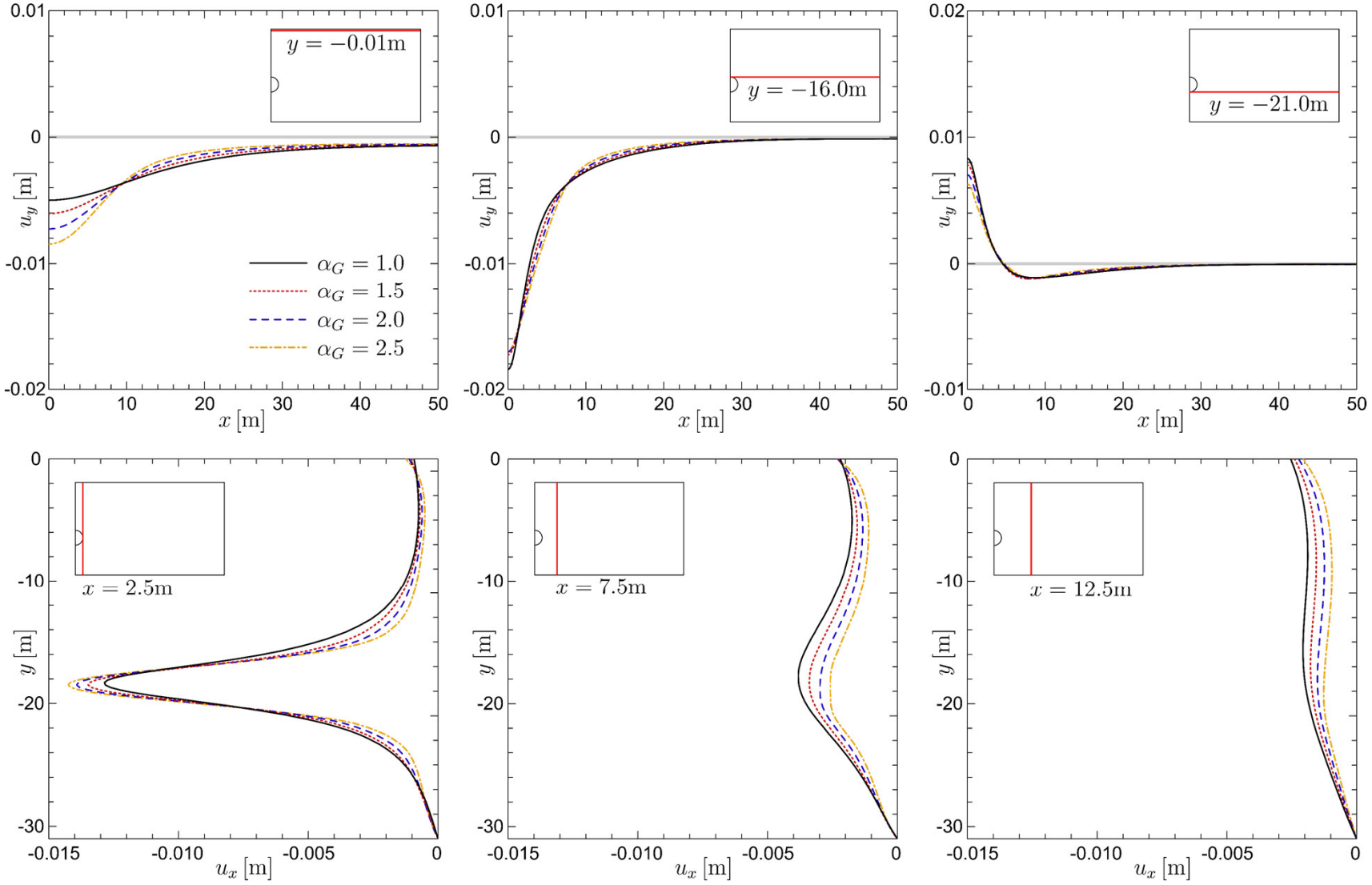

Avgerinos, Potts, Standing, 2016

# Application of the AHS-Brick model in an example of the boundary value tunneling problem in plane strain and undrained conditions




discretization, the tunnel with a diameter of 5m, the lining is subjected to contraction (volume loss) 1%, isotropic initial stress condition $K_0$=1.0

# Results of calculations, settlement troughs ($u_y$) at different depths and profiles of the horizontal component of displacement ($u_x$)



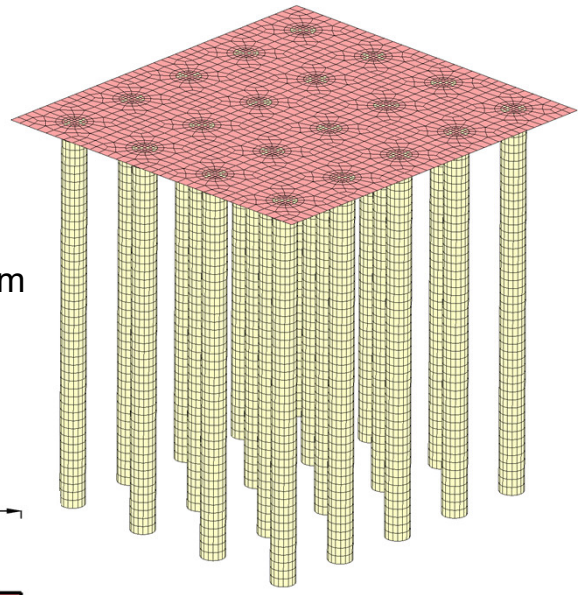
# Investigation of soil–piled raft foundation (PRF) interaction, Artur Zachodni, PhD

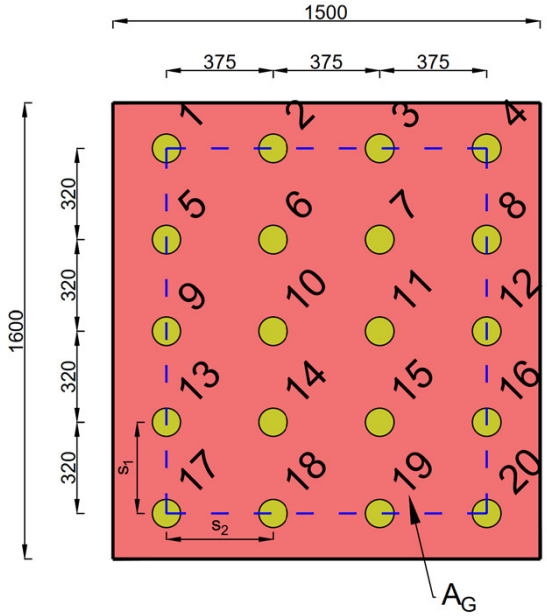
**pile grid 5 x 4**

piles $A_G$=15,7m$^2$
raft $A_f$=240m$^2$

average pile spacing $s_{avg}$=3.475m
pile diameter D=1.0m

filling factor
$(A_G/A)/(s_{avg}/D)$= 0.17



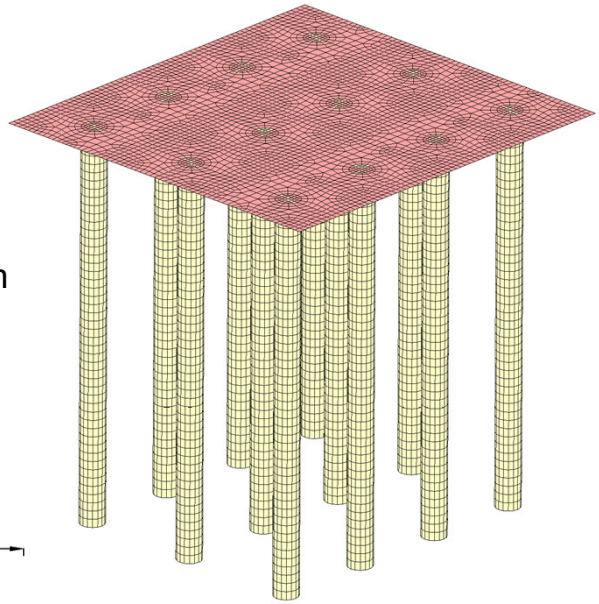


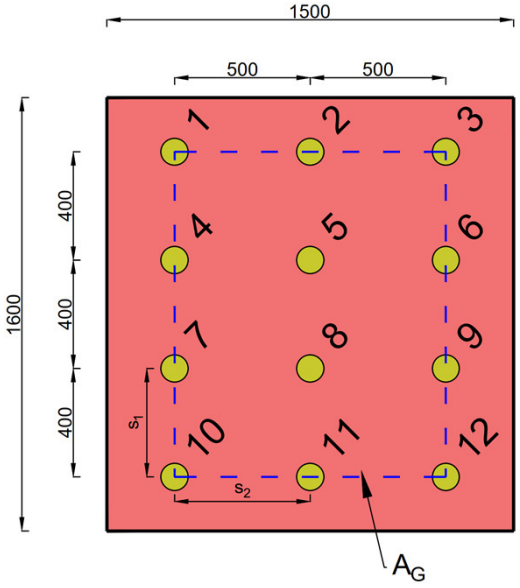

**pile grid 4 x 3**

piles $A_G$=9.42m$^2$
raft $A_f$=240m$^2$

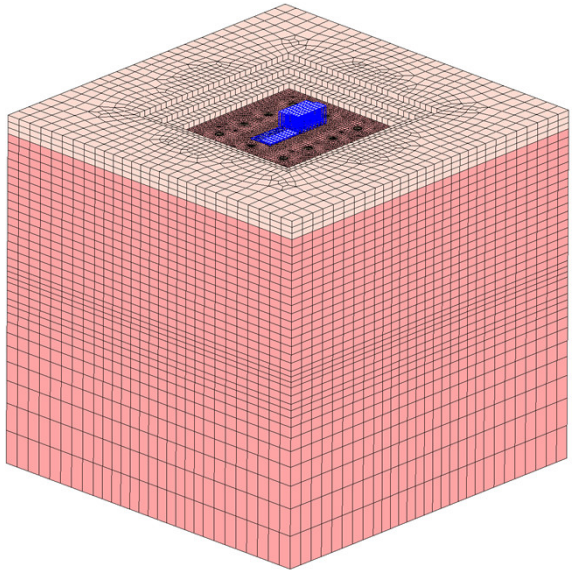
average pile spacing $s_{avg}$=4.5m
pile diameter D=1.0m

filling factor
$(A_G/A)/(s_{avg}/D)$= 0.11

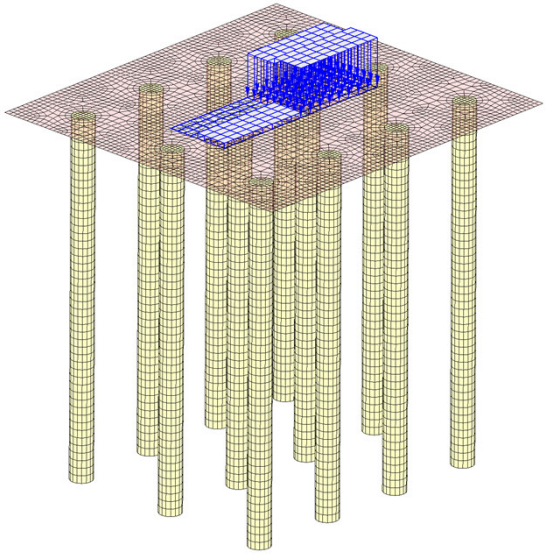




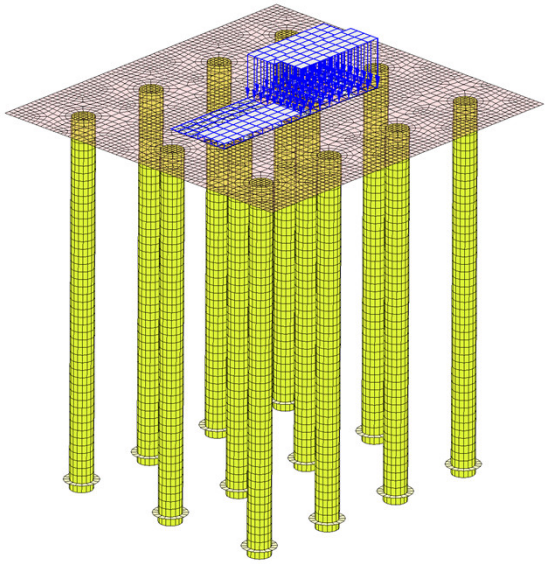
# Model details, FE discretisation



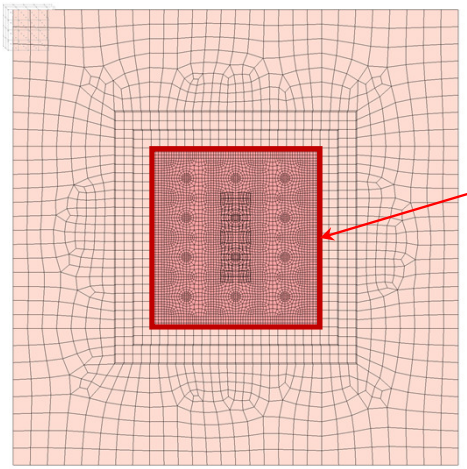
284322 octahedral element



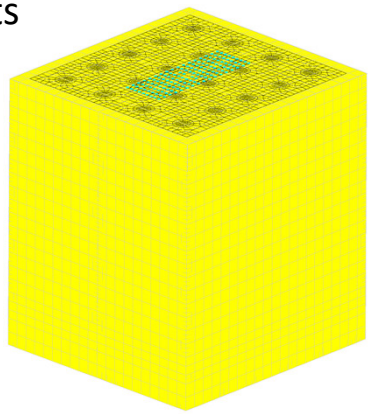
piles: continuum
raft: shell elements



9984 interface elements



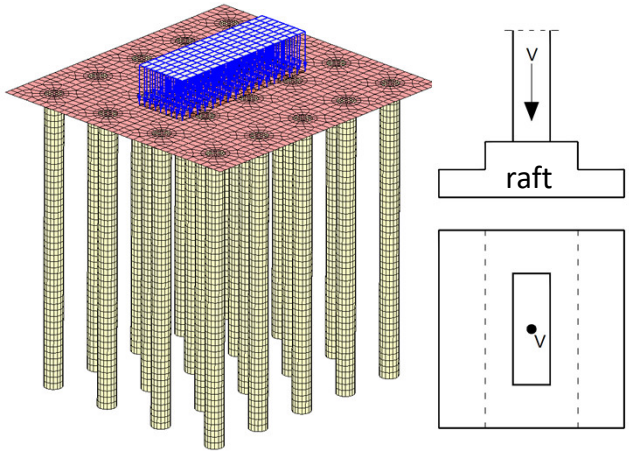
mesh tying



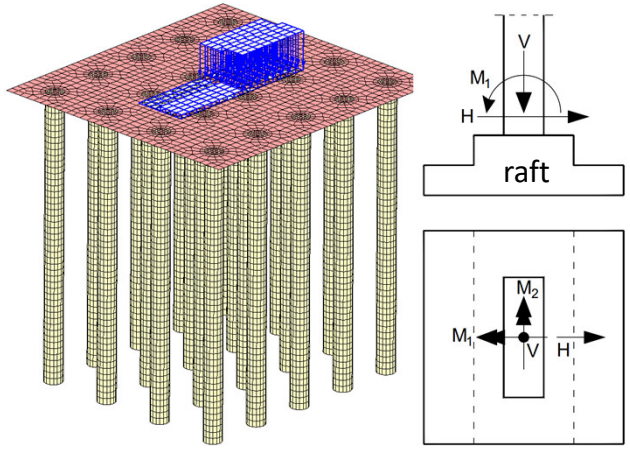
area under foundation

# Loading, time course in consolidation calculations ($k_x$=$k_y$=$k_z$=1e-4 m/d)

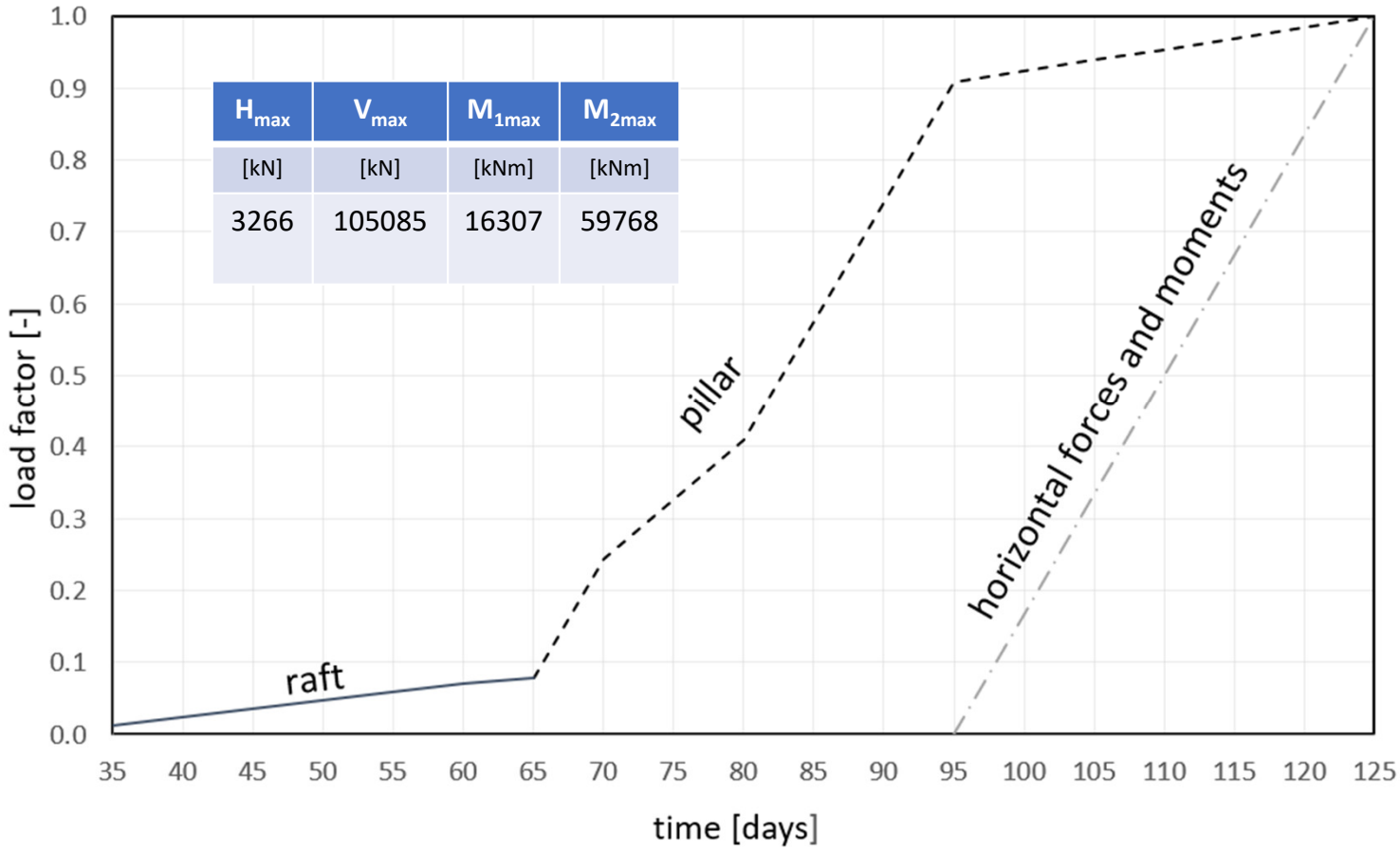
simple vertical loading




real loading (bridge foundation)
vertical and horizontal forces, moments




| $H_{max}$ | $V_{max}$ | $M_{1max}$ | $M_{2max}$ |
|---|---|---|---|
| [kN] | [kN] | [kNm] | [kNm] |
| 3266 | 105085 | 16307 | 59768 |



# Material parameters for isotropic reference model

strength:

$\phi = 27°,\ c = 20\ \text{kPa},\ \psi = 0°,\ D = 0.25,\ f_\text{t} = 0\ \text{kPa}$

barotropy:

$m = 0.7,\ p_\text{ref} = 100\ \text{kPa},\ f_\sigma = \dfrac{p}{p_\text{ref}}$

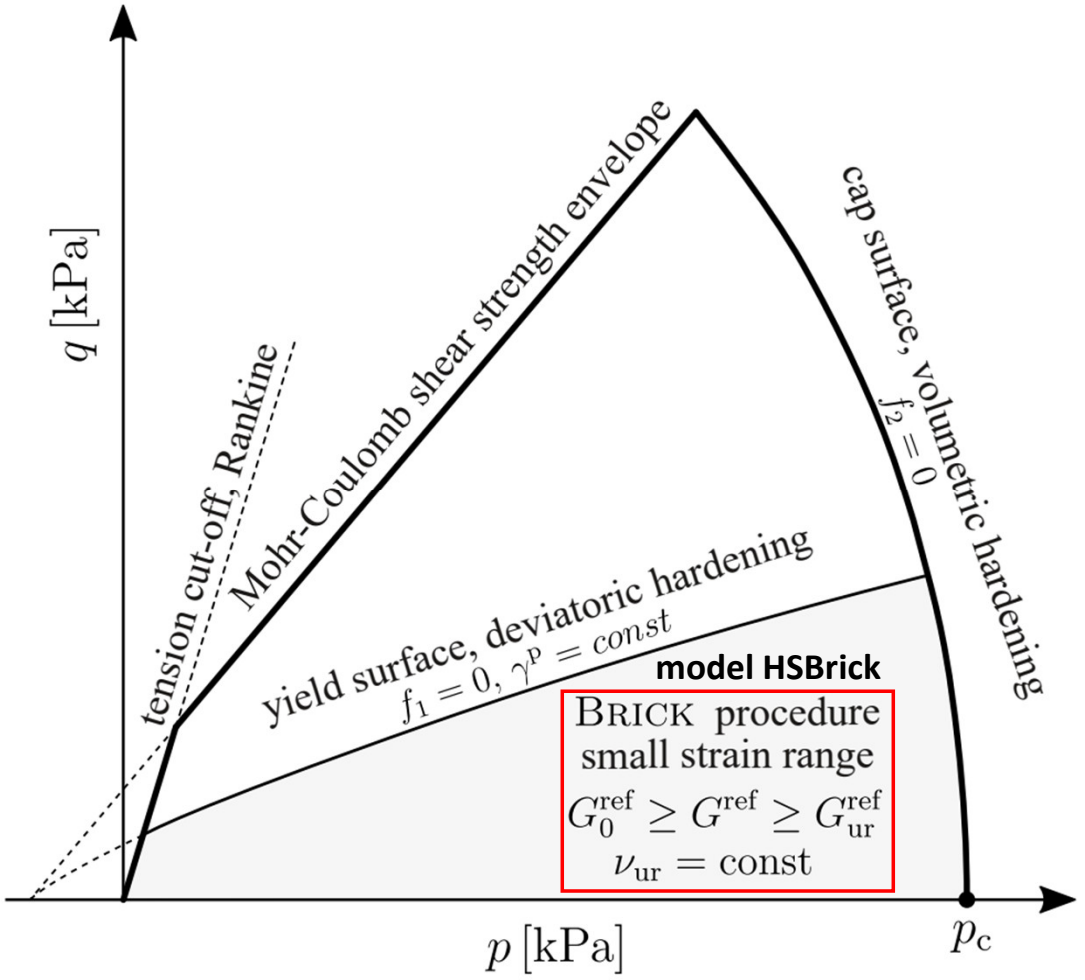
deviatoric and volumetric hardening:

$E_{50}^\text{ref} = E_\text{oed}^\text{ref} = 20000\ \text{kPa}, R_f = 0.9,\ K_0^\text{NC} = 1 - \sin(\phi)$

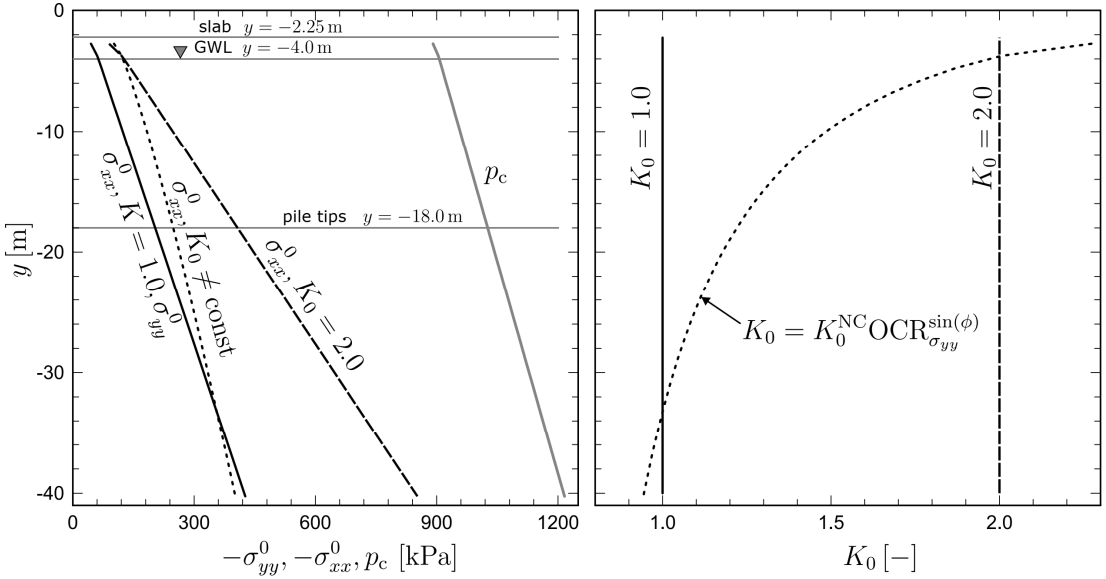
small strain paraelasticity:

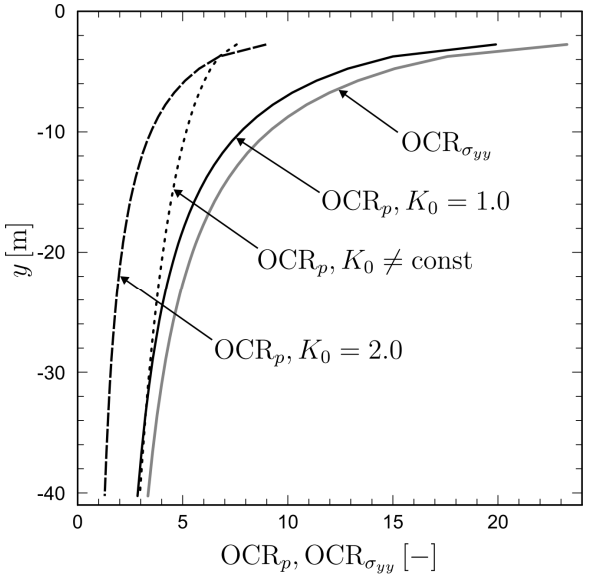
$E_0^\text{ref} = 200000\ \text{kPa}\,(G_0^\text{ref} = 80000\ \text{kPa}),\ \nu_\text{ur} = 0.25$

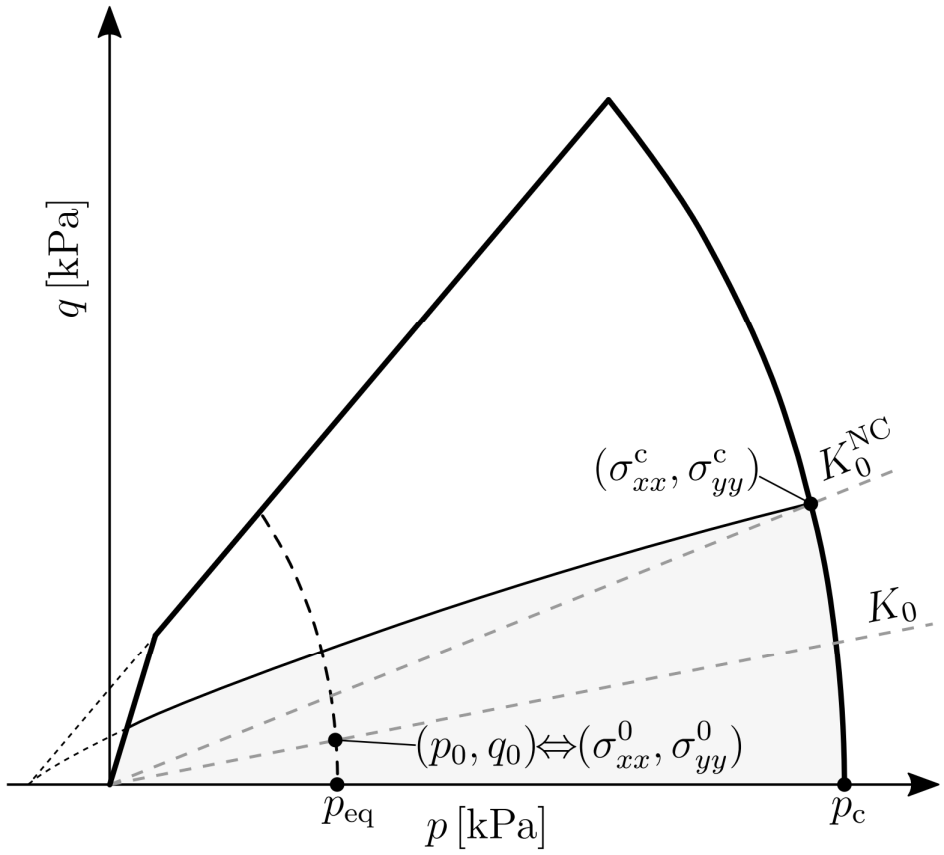
$G_\text{ur}^\text{ref} = 24000\ \text{kPa}\,(E_\text{ur}^\text{ref} = 60000\ \text{kPa}),\ \gamma_{0.7} = 0.0002$



# Initial conditions, overconsolidation ($q_{POP}$=1000 kPa) and $K_0$

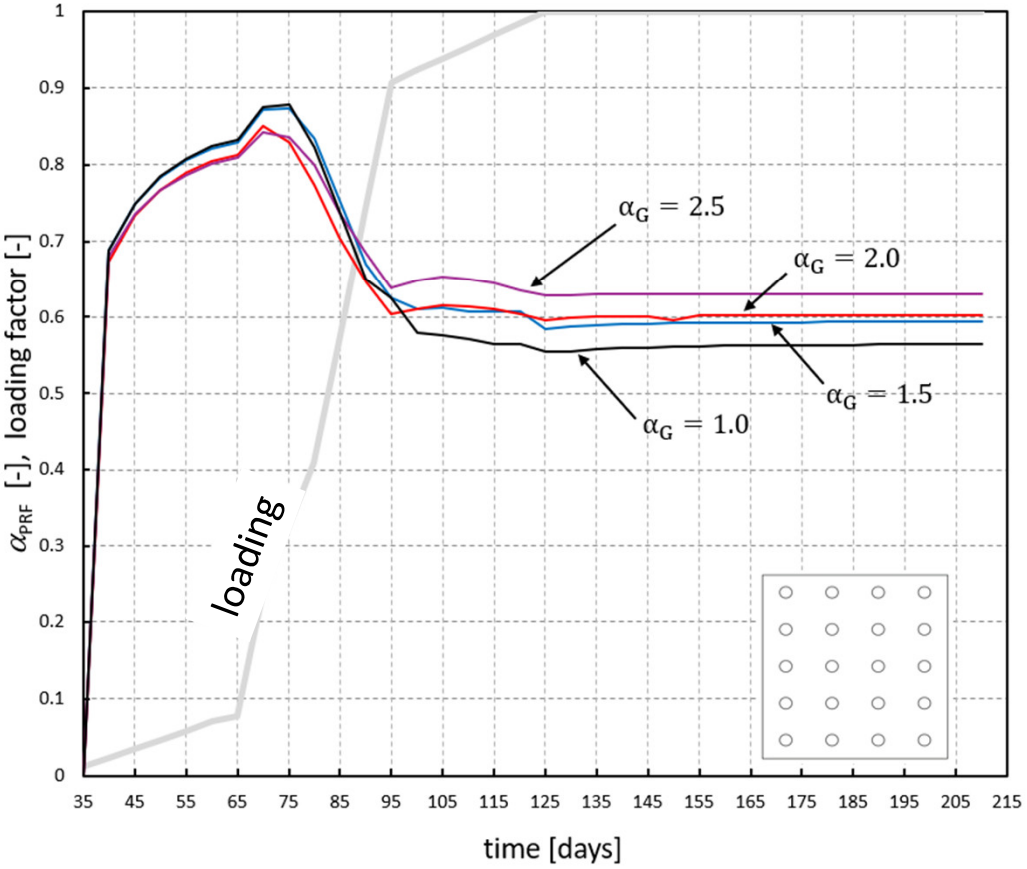


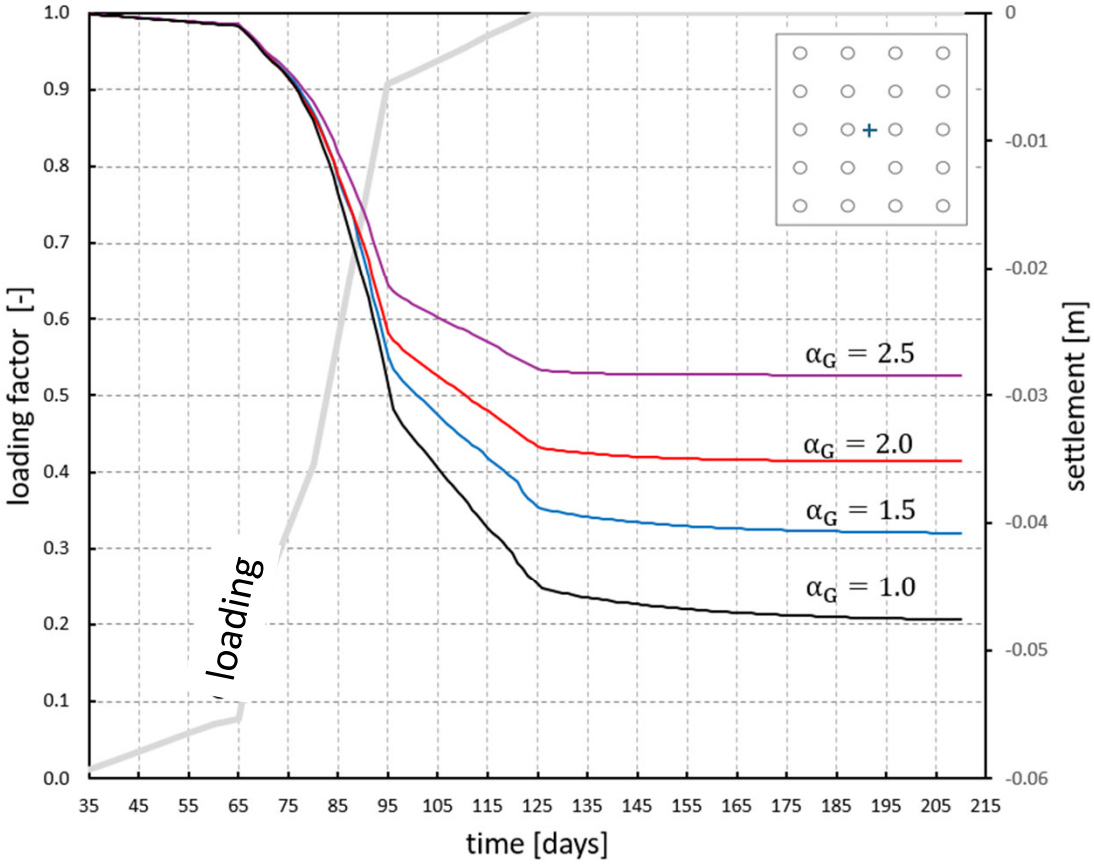




$$\mathrm{OCR}_{\sigma_{yy}} = \frac{\sigma^c_{yy}}{\sigma^0_{yy}}, \quad \mathrm{OCR}_p = \frac{p_c}{p_{eq}}$$

# Results, $\alpha_{PRF}$ and time – settlement curvrs for different $\alpha_G$ values, pile grid 5x4

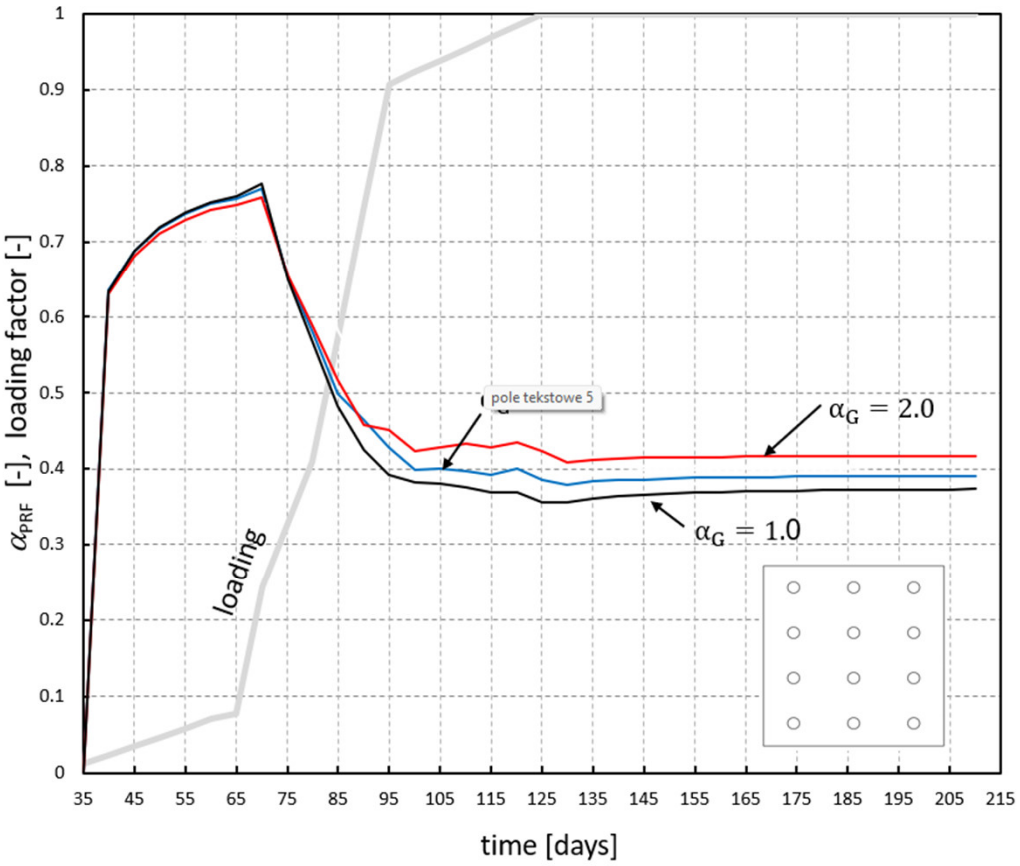


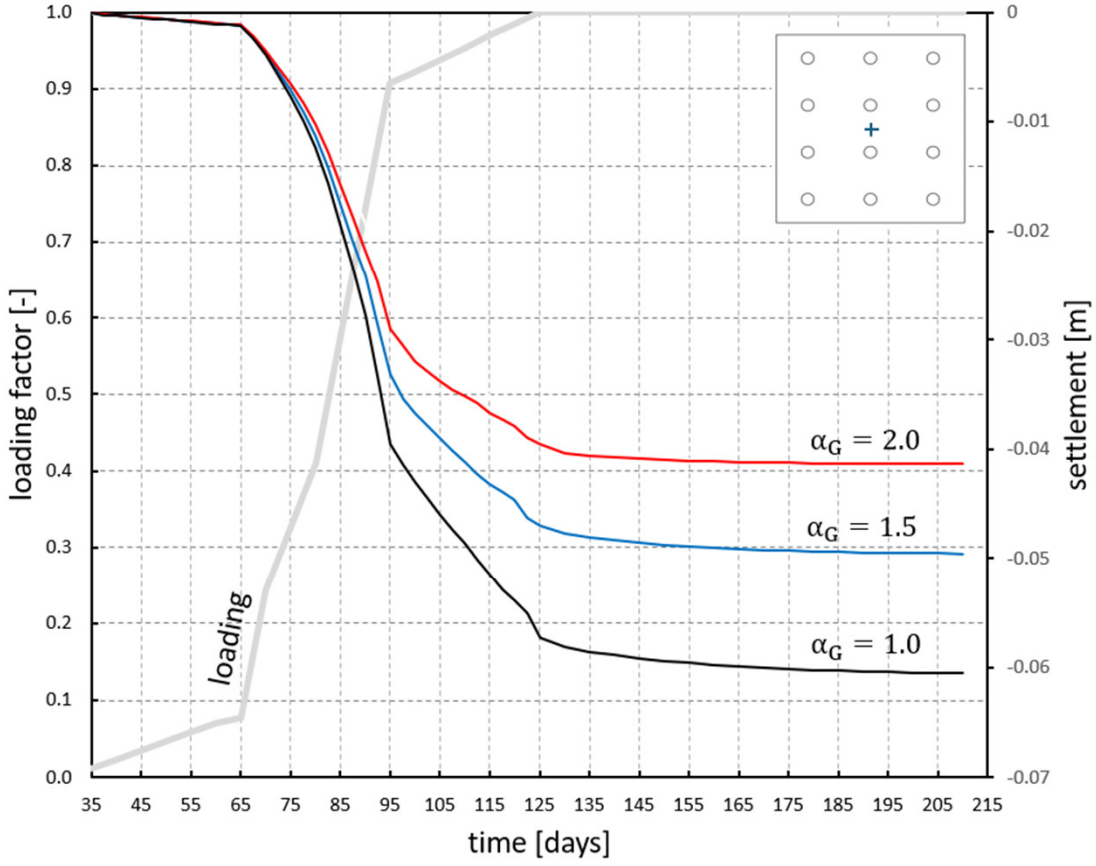




$$\alpha_{\mathrm{PRF}} = \frac{\sum_{j=1}^{m} R_{pile,k,j}(s)}{R_{tot,k}(s)}$$

$$G_{vh}^{\mathrm{ref}} = \mathrm{const},\ \nu_{vh} = const,\ \alpha_G,\ \beta = 1.6$$

# Results, $\alpha_{PRF}$ and time – settlement curvrs for different $\alpha_G$ values, pile grid 4x3




$$G_{vh}^{\mathrm{ref}} = \mathrm{const},\ \nu_{vh} = const,\ \alpha_G,\ \beta = 1.6$$

## Future tasks

- Real case studies – tunnelling, PRF, …
- Incorporation of shear strength anisotropy (AX1 scaling or Pietruszczak&Mróz)
- For rock materials softening and stiffer behaviour (e.g. no volumetric or/and deviatoric hardening)
- Another project: constitutive models for soft soils (creep, anisotropy).

**Thanks for attention**