PROBABILISTIC VS. DETERMINISTIC DESIGN IN EUROCODES 2.0, WITH POTENTIAL ECONOMIC GAIN

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Outline

- Problem description
- Deterministic approach, with respect to SIA 267 (Swisscodes)
- FE model and validation
- Probabilistic approach
  - Reference case: compute G_k knowing width b
  - What if COV >> ?
  - What if more lab tests (Bayesian approach) ?
  - Inverse problem: design width b knowing G_k
- Conclusions

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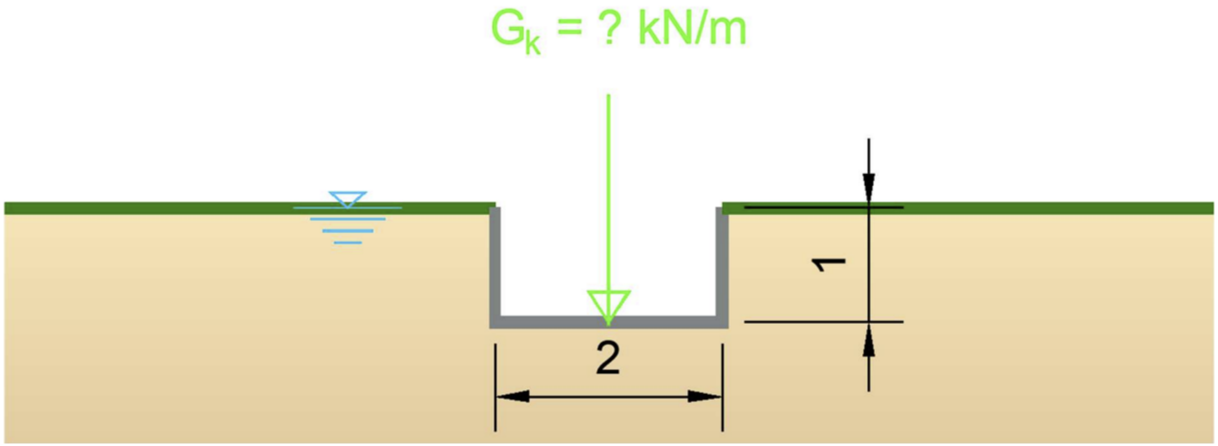
Problem description

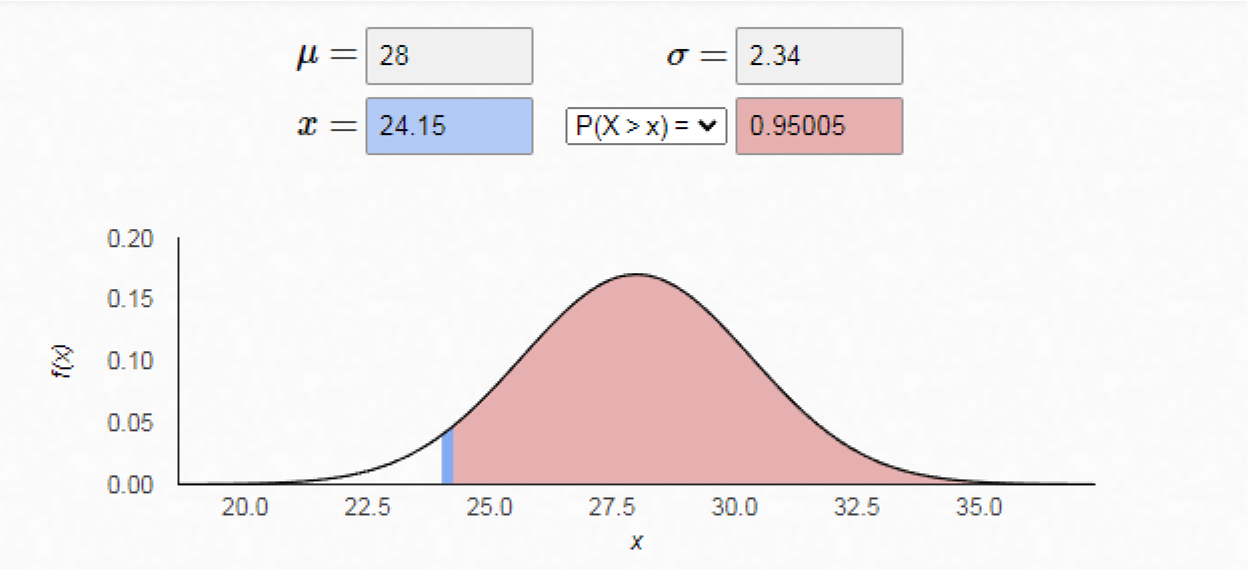
- Consider the following spread continuous footing, with b = 2 m
- Given soil's strength parameters c' and ϕ'
- What is the load G_k that we can put on the footing?



[Labiouse, Kohler 2025]

Problem description

- Assume a deterministic value for c'_k = 5 kPa
- Assume the following normal distribution for ϕ'
  - ϕ'_m = 28°, with a standard deviation of 2.34° (COV = 8.4%)
  - this yields ϕ'_k = 24.15° (5% fractile)



Problem description

- Typical COV for ϕ' are given in the 2024 report **https://eurocodes.jrc.ec.europa.eu/publications/reliability-based-verification-limit-states-geotechnical-structures**

- COV = 8.4% falls in the low property variability, typical of good-quality direct laboratory or field measurements

Geotechnical parameter	Property variability	COV (%)
Effective stress friction angle	Low[1]	5-10
	Medium[2]	10-15
	High[3]	15-20
Horizontal stress coefficient	Low[1]	30-50
	Medium[2]	50-70
	High[3]	70-90

(1) Typical of good-quality direct laboratory or field measurement.
(2) Typical of indirect correlations with good field data, except for the SPT.
(1) Typical of indirect correlations with SPT field data and with strictly empirical correlations.

Source: Phoon and Kulhawy 2008

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Deterministic approach

- Based on SIA 267 (SwissCodes), following EC7-3 and Terzaghi

$$R_N = A'(c'N_c + q'N_q + 0.5\gamma^* B' N_\gamma)$$

$$N_q = e^{\pi tan\phi'} tan^2\left(45 + \frac{\phi'}{2}\right)$$
$$N_c = (N_q - 1)\, cot\phi'$$
$$N_\gamma = 1.8\left(N_q - 1\right) tan\phi'$$

- Application of resistance partial factors
  - $c'_d = c'_k / 1.5 = 3.33$ kPa
  - $\tan(\phi_d) = \tan(\phi_k) / 1.2 \Rightarrow \phi_d = 20.49°$
- This yields $R_{Nd} = 271$ kN/m'
- Assuming a load partial factor $\gamma_F = 1.35$ =>

SIA	
Phi_k [°]	24.15
Phi_d [°]	20.49
Phi_d [rad]	0.4
c_k [kPa]	5.0
c_d [kPa]	3.3
N_q	6.7
N_c	15.3
N_gamma	3.8
q'_pd [kPa]	135.5
R_Nd [kN/m']	271.0
gamma_F	1.35
G_k [kN/m']	200.8

[Labiouse, Kohler 2025]

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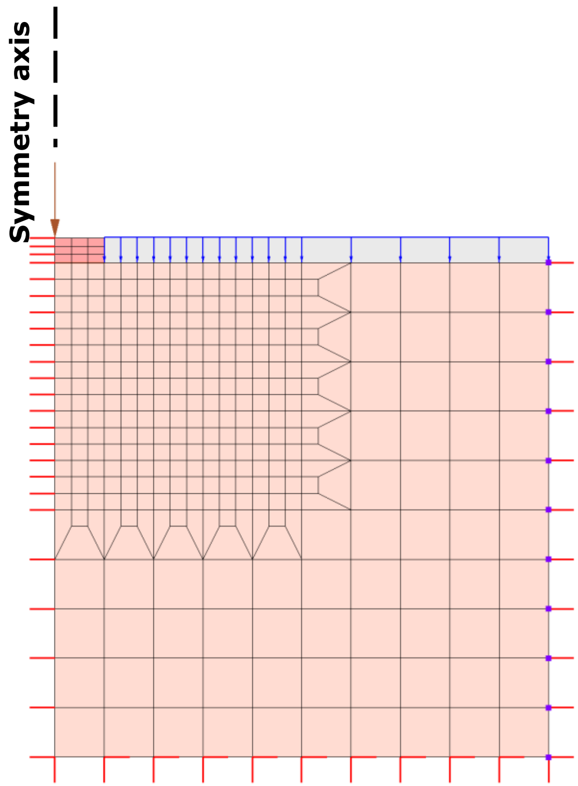
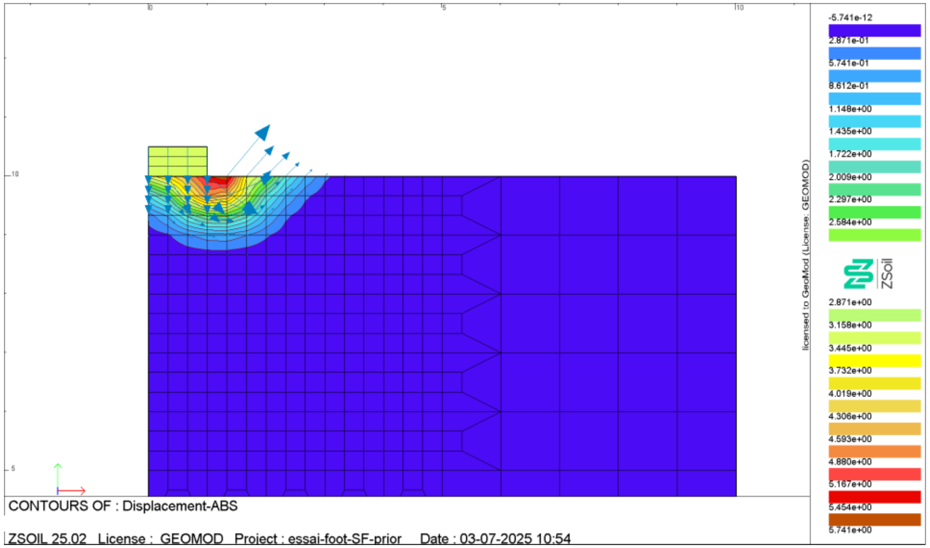
FE model and validation

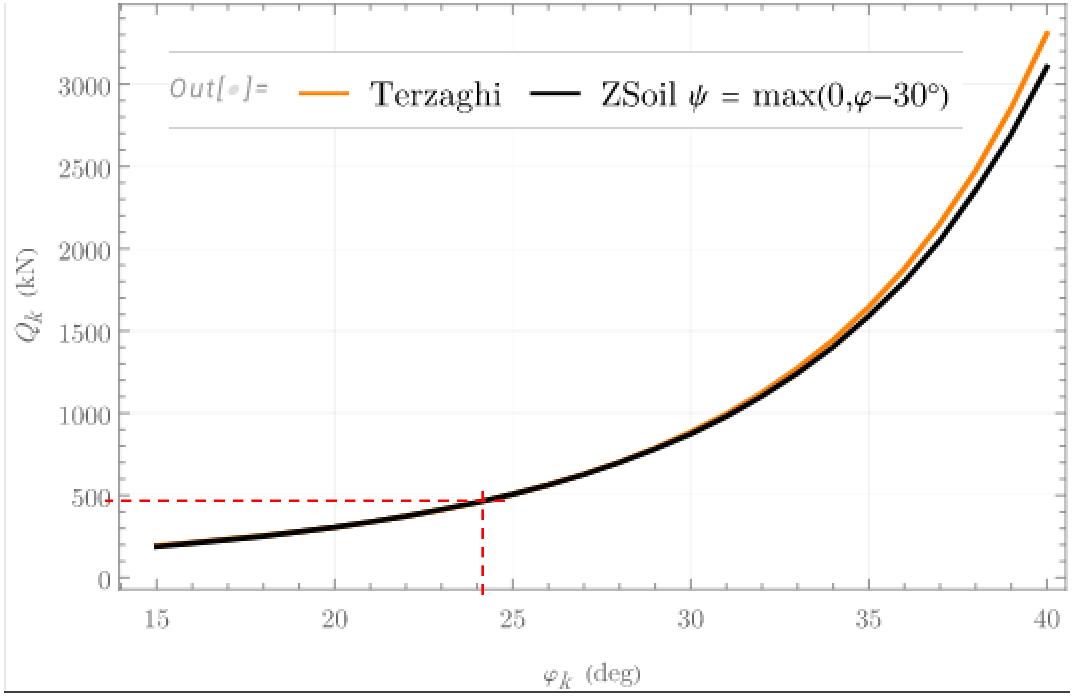
- The following 2D plane strain mesh is used with ZSoil v25.02
- The load is applied step by step, until divergence occurs





FE model and validation

- Bearing capacity (here without any partial factors) obtained with ZSoil is shown to match the Terzaghi manual approach, for a large range of friction angles
- In particular, R_N = 464.4 kN/m' for ϕ'_k = 24.15° and c'_k = 5 kPa



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Probabilistic approach
Reference case: compute G_k knowing width b

- How to switch from deterministic to probabilistic framework?

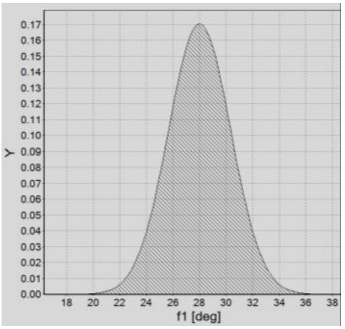
- Methodology:

  - Apply load R_N = 464.4 kN/m' (ϕ'_k = 24.15°, c'_k = 5 kPa)
  - Introduce ϕ' PDF = Gaussian (28°, 2.34°)
  - Compute probability of failure P_f (safety factor < 1.0)
  - Compare with target reliability values, given in https://eurocodes.jrc.ec.europa.eu/publications/reliability-based-verification-limit-states-geotechnical-structures

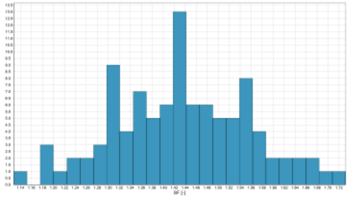
Consequence class	1-year reference period β	50-year reference period	
		β	$P_{f,50}$
CC3	5.2	4.3	~ 10^{-5}
CC2	4.7	3.8	~ 10^{-4}
CC1	4.2	3.3	~ 10^{-3}

Source: EN 1990-1

Probabilistic approach
Reference case: compute G_k knowing width b

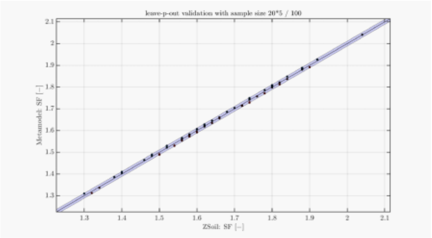


- Define ϕ' PDF in ZSoil postprocessor

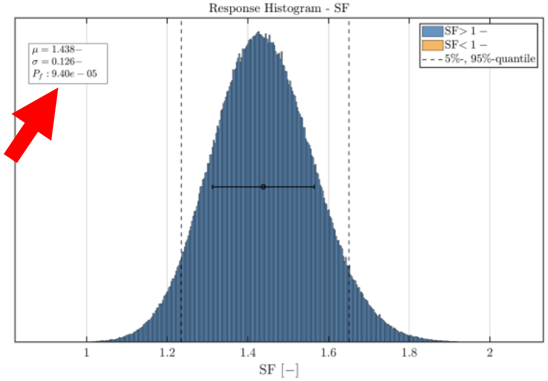


- Compute experimental design:

  100 samples to evaluate SF



- Generate meta-model (PCE) for SF



- Perform reliability analysis:

  P_f (SF<1.0) = 9.4e-5

Probabilistic approach
Reference case: compute G_k knowing width b

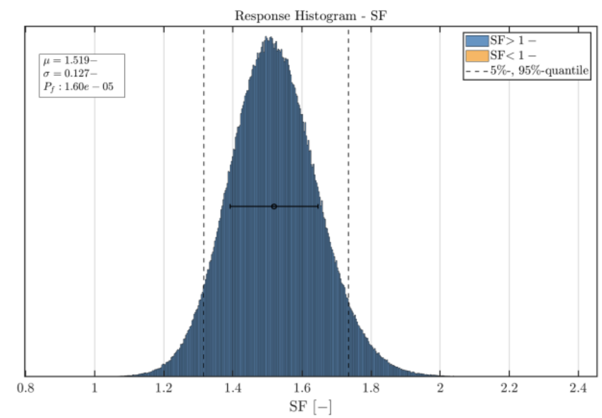
- P_f < 1e-4 ✓ so G_k = 464.4 kN/m' / 1.35 = 344 kN

- **72% gain** with respect to G_k(deterministic) = 200.8 kN/m' !

- Performing a full probabilistic analysis with

  - ϕ' PDF = Gaussian (mean = 28°, COV = 8.35%)
  - c' PDF = Gaussian (mean = 7.45 kPa, COV = 20%)
  - G_k PDF = Gaussian (mean = 464.4 kN/m', COV = 5%)

Leads to P_f = 1.6e-5



- Amazing! But...

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Probabilistic approach
What if COV >> ?

- Imagine COV is doubled
- ϕ' PDF = Gaussian (mean = 28°, COV = 16.7%) => ϕ'_k = 20.3°
- Deterministic design yields G_k = 143.9 kN/m'
- Probabilistic design with **R_N = 316.3 kN/m'**:

Pf = 4e-3 => no gain !

- Importance of lab tests to lower COV...

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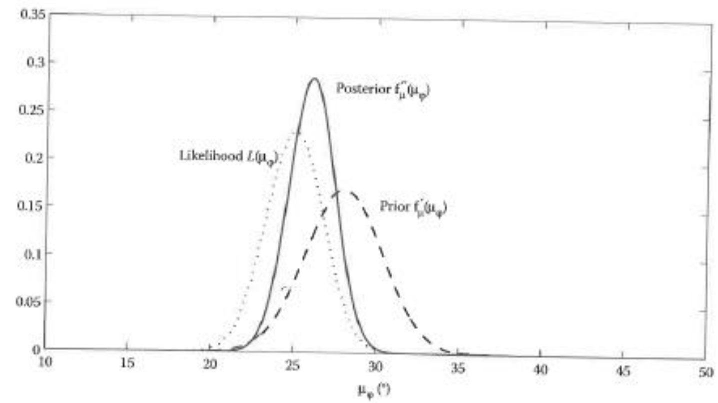
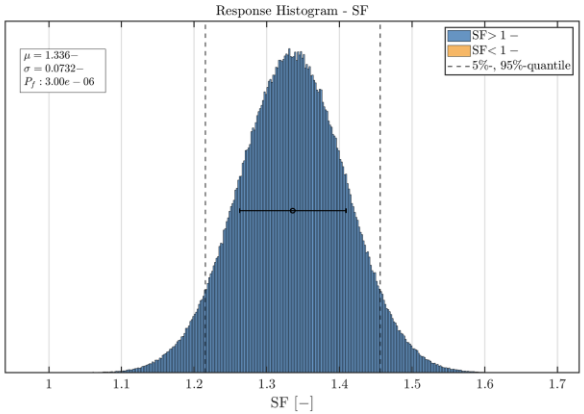
Probabilistic approach
What if more lab tests?

- Imagine we have three additional shear tests at hand, leading to:
  - $\phi'_1 = 25.6°$, $\phi'_2 = 25.5°$, $\phi'_3 = 24°$
- Using Bayesian updating, the posterior distribution for ϕ' will be: Gaussian (mean = 26.08°, COV = 5.3%)
- Note that COV <<, but also mean < !
- Performing the same analysis as before, we get P_f = 3e-6 << 1e-4 ✓
- With b = 1.8 m (instead of 2 m), we get P_f = 1.7e-4 ≅ 1e-4 (10% gain on concrete)



Straub D., Papaioannou I.: Bayesian analysis for learning and updating geotechnical parameters and models with measurements – 5. In: Risk and Reliability in Geotechnical Engineering (eds. Phoon K.-K., Ching J.), CRC Press, 2015



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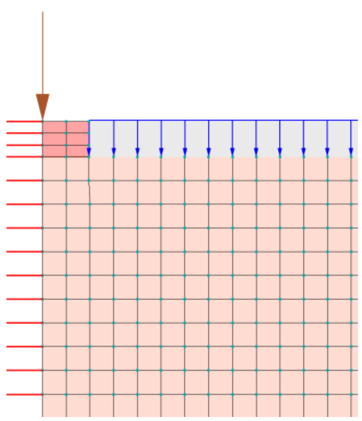
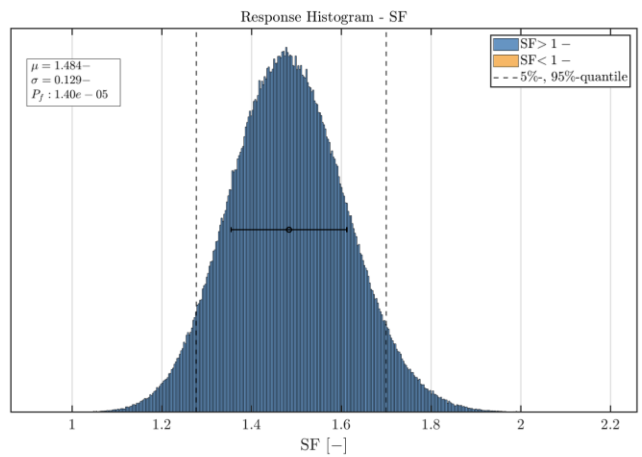
Probabilistic approach
Inverse problem: design width b knowing G_k

- Methodology:
  - Apply original load G_k x 1.35 = 200.8 x 1.35 = 271 kN/m’
  - Introduce ϕ' PDF = Gaussian (28°, 2.34°)
  - Find b such as Pf < 1e-4
  - For b = 1.35 m (instead of 2 m), Pf = 1.4e-5 << 1e-4 ✓





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Conclusions

- For b = 2 m, with deterministic design (SIA 267), and $\phi'_k = 24.15°$

  G_k = 200 kN/m′, G_d = 270 kN/m′

- Same, with probabilistic design, $\phi'_m = 28°$ and COV = 8.35%

  G_k = 344 kN/m′, G_d = 465 kN/m′ (**72% gain on load**)

- But, with probabilistic design, $\phi'_m = 28°$ and COV = 16.7%

  NO GAIN

- Same, after a few (bad !) tests, $\phi'_m = 26.08°$ and COV = 5.3%

  b = 1.8 m for G_d = 464.4 kN/m′ (10% gain on concrete)

Conclusions

- Alternatively: probabilistic design with G_d = 270 kN/m'

  b = 1.35 m (**33% gain on concrete**)

- The lower COV... the more benefits => **testing is always good and can lead to significant gains**

- Thank you to V. Labiouse, D. Kohler (HEIA-FR)

- Questions?