Conservative Ultimate Limit Load Estimation of Geotechnical Systems Using the FEM

GEOMOD

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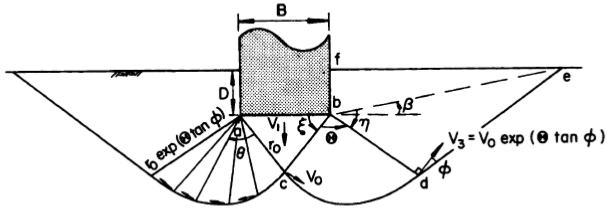
THEORY

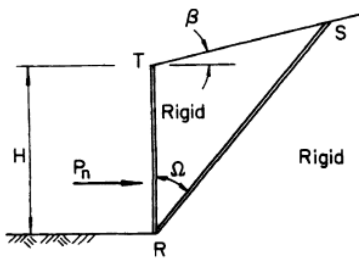
STABILITY PROBLEMS

Bearing capacity



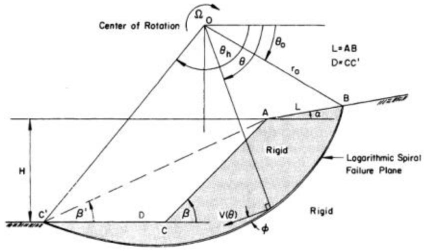
[1]

Active/Passive earth pressure



[1]

Slope stability



[1]

- On the material side determined by its failure behaviour
  - Most often modelled with Mohr-Coulomb failure criterion → φ, c, ψ

- Suitable framework for solving such problems: **Limit Analysis**

[1] Chen, W.-F. (1975) *Limit Analysis and Soil Plasticity.* Amsterdam: Elsevier Scientific Publishing Co.

Limit Analysis

Kinematic method:

- Solve energy balance on failure mechanism

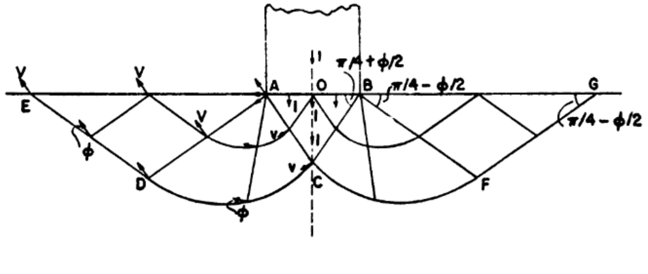


FIG. 6. Prandtl Solution. [1]

Static method:

- Find allowed stress field that is in static equilibrium

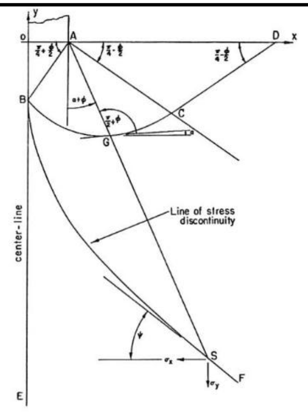
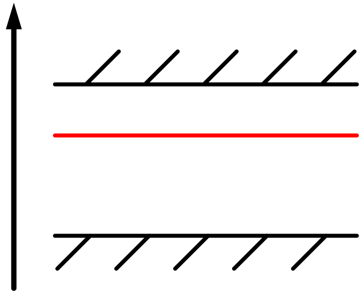


FIG. 1 AN EXTENSION OF PRANDTL STRESS SOLUTION FOR A FOOTING ON A SOIL [2]

The solutions from both methods bound the true failure load from above and below:

Failure load



Upper bound (e.g. kinematic method)

Exact solution

Lower bound (e.g. static method)

$\boldsymbol{\varphi} = \boldsymbol{\psi}$ (→ AFR = *associated flow rule*) is required!

[1] Shield, R.T. (1953) 'Mixed boundary value problems in soil mechanics', *Quarterly of Applied Mathematics*, 11(1), pp. 61–75.
[2] Shield, R.T. (1954) 'Plastic Potential Theory and Prandtl Bearing Capacity Solution', *Journal of Appl. Mech.*, 21(2), pp. 193–194.

Limit Analysis

F_u is independent of the pre-failure behaviour of the system [1]
(i.e. E, ν, hardening, construction stages, load path)

Only input: φ, c

Limit Analysis ($\psi = \varphi$!)

Good analytical solutions

can be complex to find	easy once found

Numerical tools:
- FELA (Finite Element Limit Analysis)
- FEM (displacement based → e.g. ZSoil)

Catch? → Real soils have $\psi(\dot{\varepsilon}^p) < \varphi$ → ~~Limit Analysis~~

[1] Chen, W.-F. (1975) *Limit Analysis and Soil Plasticity.* Amsterdam: Elsevier Scientific Publishing Co.

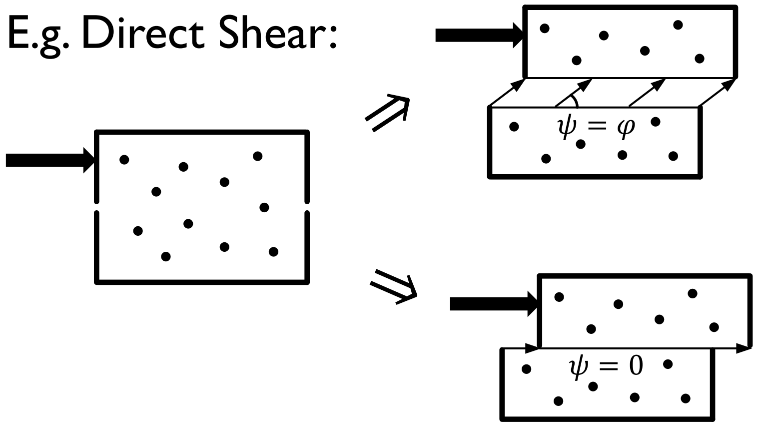
$\psi < \varphi$

$\psi < \varphi$ → $F_{ult}^{\psi<\varphi} \leq F_{ult}^{\psi=\varphi}$ — Why?

2) <u>Localized failure planes</u> may occur, with an apparent softening at the time of their emergence [1,2].

1) Systems may present <u>resistance to volume increase</u>.

E.g. Direct Shear:

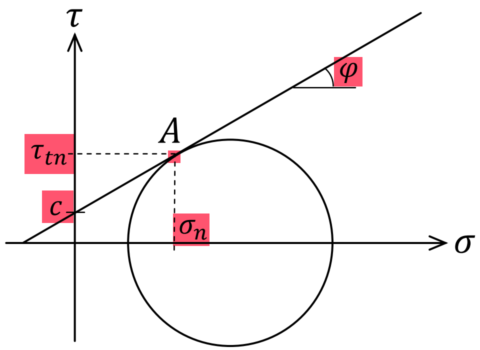


[1] Davies, EH. (1968) 'Theories of plasticity and the failure of soil masses', In *Soil Mechanics: Selected Topics*, Lee IK (ed.), Butterworth: London, pp. 341-380.
[2] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *International Journal for Numerical Methods in Engineering*, 90(9), pp. 1089–1117. Available at: https://doi.org/10.1002/nme.3358.

$\psi < \varphi$ - 2) Localized failure planes may occur, with an apparent softening at the time of their emergence.

Immediately pre-yield: (Point A)

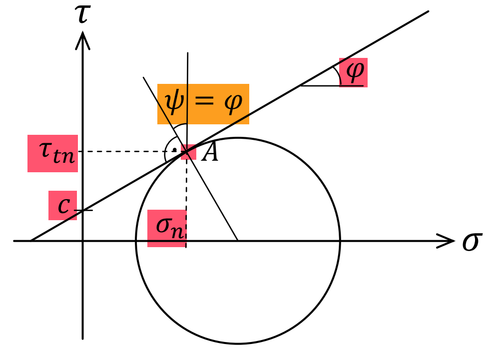
$\tau_{tn} = \boldsymbol{tan\,\varphi} \cdot \sigma_n + \boldsymbol{c}$



Associated flow rule $\psi = \varphi$:

Post-yield: (Point A)

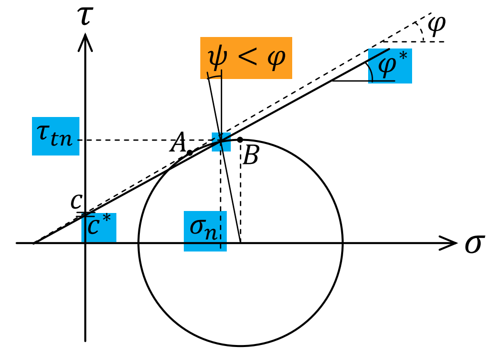
$\tau_{tn} = \boldsymbol{tan\,\varphi} \cdot \sigma_n + \boldsymbol{c}$

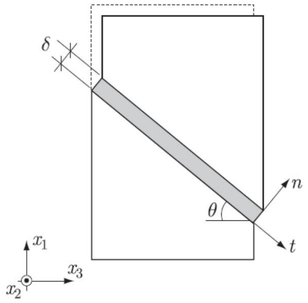


Nonassociated flow rule $\psi < \varphi$:

Post-yield: (between points A and B)

$\tau_{tn} = \boldsymbol{tan\,\varphi}^* \cdot \sigma_n + \boldsymbol{c}^*$





Localized failure plane in t-direction [1]

[1] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *International Journal for Numerical Methods in Engineering*, 90(9), pp. 1089–1117. Available at: https://doi.org/10.1002/nme.3358.

$\psi < \varphi$ - 2) Localized failure planes may occur, with an apparent softening at the time of their emergence.

- M.C. yield surface: $F = \frac{J_1}{3}\sin\varphi - \sqrt{J_{2D}}\cos\theta - \frac{\sqrt{J_{2D}}}{\sqrt{3}}\sin\varphi\sin\theta + c\cos\varphi = 0$ or $\tau_{tn} = \boldsymbol{\tan\varphi}\cdot\sigma_n + \boldsymbol{c}$
- M.C. plastic potential: $G = \frac{J_1}{3}\sin\psi - \sqrt{J_{2D}}\cos\theta - \frac{\sqrt{J_{2D}}}{\sqrt{3}}\sin\psi\sin\theta + const. = 0$ $\rightarrow$ in general: $\dot{\varepsilon}_n^p = \tan\psi\cdot\dot{\gamma}_{tn}^p$

- Assumption that localization happens $\Rightarrow \delta \rightarrow 0$
- $\Rightarrow \dot{\varepsilon}_t^p \ll \dot{\varepsilon}_n^p \Rightarrow \dot{\varepsilon}_t^p \rightarrow 0$
- $\dot{\varepsilon}_t^p = \dot{\lambda}\frac{\partial G}{\partial\sigma_t} = 0 \Rightarrow \cdots$

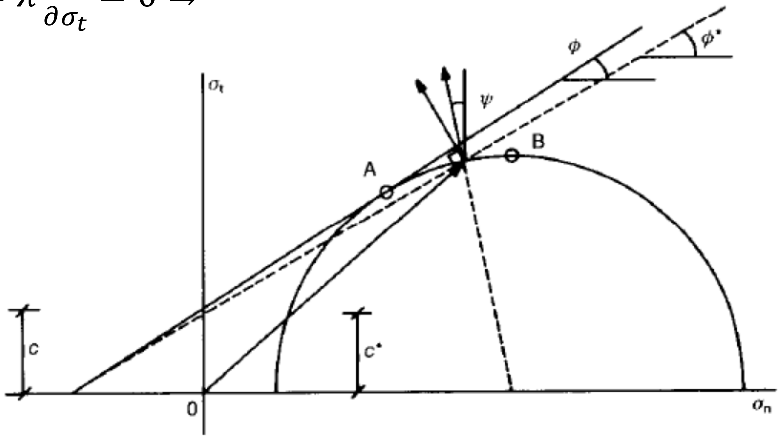


Fig. 8. Traction and velocity jumps for a linear Mohr–Coulomb yield condition and a coaxial non-associative flow rule [2]

$\Rightarrow$ In localized yield state: [1]

$\tau_{tn} = \boldsymbol{\tan\varphi^*}\cdot\sigma_n + \boldsymbol{c^*}$

with:

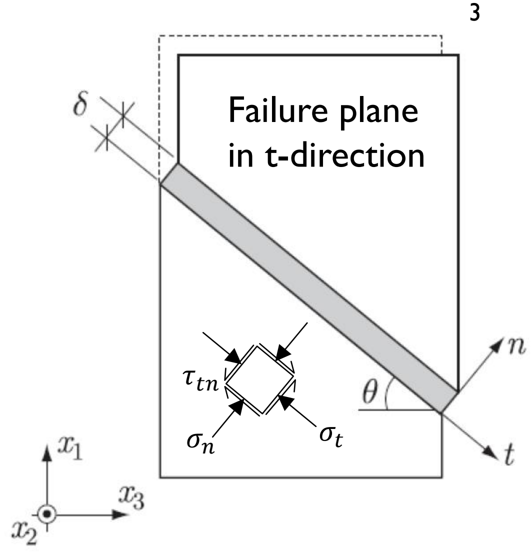
$c^* = \beta c$

$\tan\varphi^* = \beta\tan\varphi$

$\beta = \frac{\cos\psi\cos\varphi}{1-\sin\psi\sin\phi} < 1$

$\Rightarrow$ apparent softening in the failure plane

Softening leads to localization ✓



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[3] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *Int. J. for Num. Meth. in Eng.*, 90(9), pp. 1089–1117.

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→

- Limit Analysis is not conservative!
- FEM OK? (Yes and no)
  - HSs, adequate initial state, construction stages
  - c–tanφ reduction

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[2] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *International Journal for Numerical Methods in Engineering*, 90(9), pp. 1089–1117. Available at: https://doi.org/10.1002/nme.3358.

$\psi < \varphi$

FEM with $\psi < \varphi$ →

Possibility of localization?
Mesh size&orientation dependency

Coarse mesh → No localization possible → Not conservative!

Fine mesh → Localization is possible →

Mathematical non-uniqueness
- Multiple possible F_u
- Solver oscillations
- Compromised convergence

Mesh orientation dependency

Lowest F_u found?

Strategies:

1. Many models → no guarantee for lowest F_u
2. Drescher and Detournay (1993)[1] → Davies' reduced strength parameters[2] together with AFR ($\psi = \varphi$). Then do Limit Analysis, FELA, FEM etc.

[1] Drescher, A. and Detournay, E. (1993) 'Limit load in translational failure mechanisms for associative and non-associative materials', *Geotechnique*, 43(3), pp. 443-456.
[2] Davies, EH. (1968) 'Theories of plasticity and the failure of soil masses', In *Soil Mechanics: Selected Topics*, Lee IK (ed.), Butterworth: London, pp. 341-380.

Reduced Strength Parameter Approach

Use Mohr-Coulomb model
with φ^*, c^*, ψ^*

$\tan\varphi^* = \beta \tan\varphi$ $\quad c^* = \beta \cdot c$ $\quad \psi^* = \varphi^*$

$$\beta = \frac{\cos\psi \cos\varphi}{1-\sin\psi \sin\phi} < 1$$

Strength in failure planes with φ^*, c^*, ψ^* = Strength in localized failure planes with φ, c, ψ

plus all benefits of the AFR

Overestimates plastic volume increase

↓

Overestimates collapse load of kinematically constrained systems

- Definition of «*degree of kinematic constraint*»?
- Amount of influence for different system types?

Examples from Literature

LITERATURE EXAMPLE 1: BIAXIAL TEST [1]

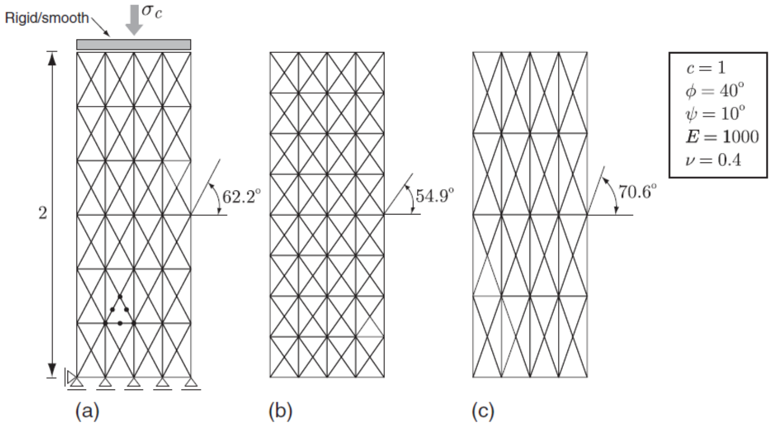


Figure 7. Biaxial test: optimal (a) and suboptimal (b, c) mesh arrangements for $\phi = 40°$ and $\psi = 10°$ (coarse meshes).

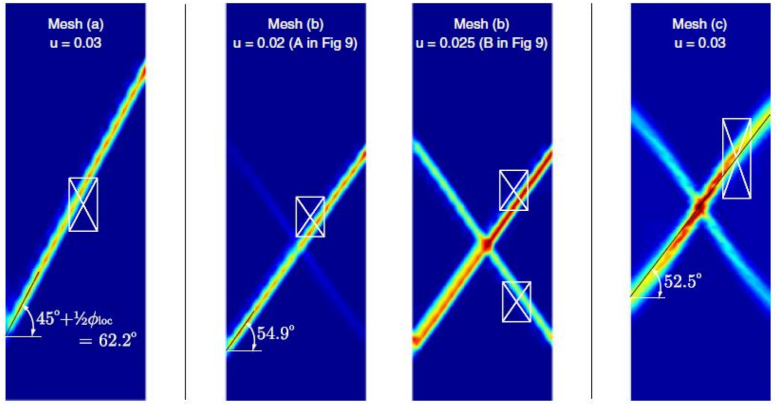


Figure 10. Biaxial test: rate of plastic shear strain for meshes (a) and (b). The shear bands follow the diagonals of the meshes in cases (a) and (b), but not in case (c).

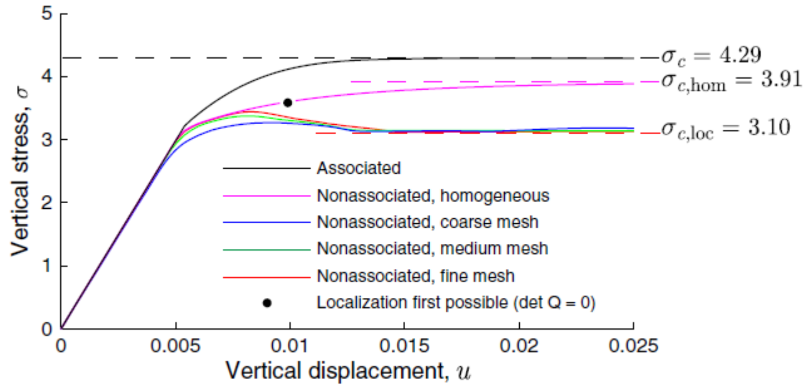


Figure 8. Biaxial test: solutions for coarse, medium, and fine meshes of type (a).

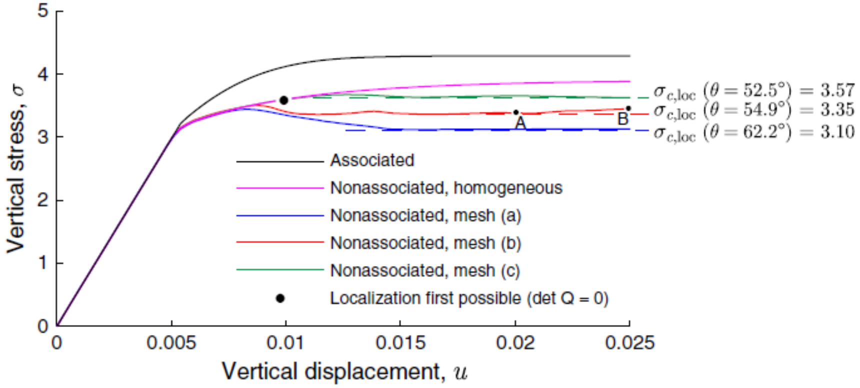


Figure 9. Biaxial test: solutions for fine meshes of types (a), (b), and (c).

[1] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *International Journal for Numerical Methods in Engineering*, 90(9), pp. 1089–1117.

LITERATURE EXAMPLE 2: STRIP FOOTING [1]

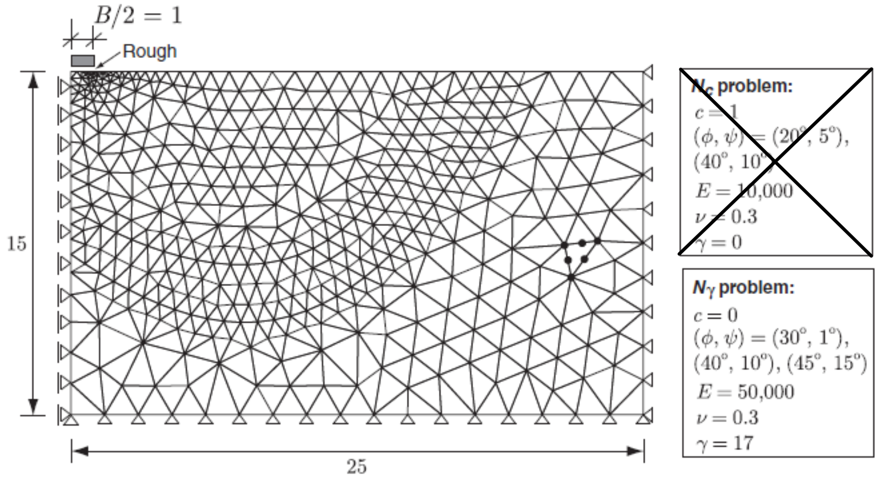


Figure 11. Strip footing: problem setup and finite element mesh (coarse).

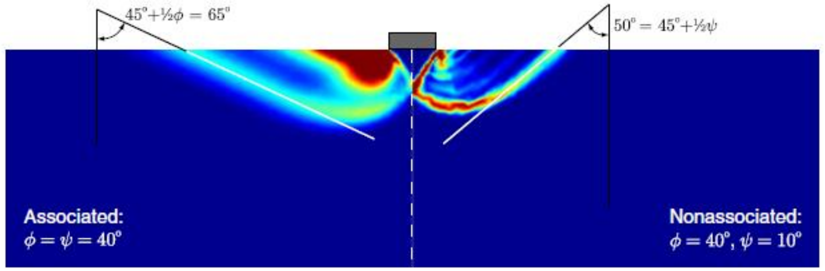


Figure 16. N_γ problem: rates of plastic shear strain at $u = 0.5$ for $\phi = 40^\circ$ and $\psi = 10^\circ$.

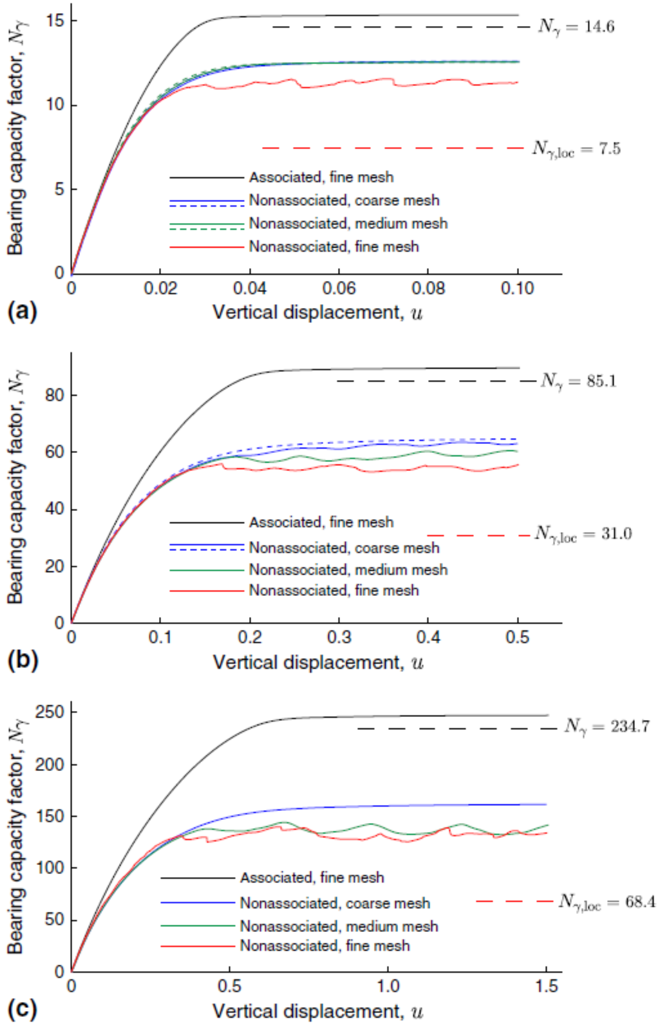


Figure 15. N_γ problem: load–displacement curves for (a) $\phi = 30^\circ$, $\psi = 1^\circ$, (b) $\phi = 40^\circ$, $\psi = 10^\circ$, and (c) $\phi = 45^\circ$, $\psi = 15^\circ$. Dashed curves correspond to the standard implicit scheme and full curves to the new apparent cohesion scheme.

[1] Krabbenhoft, K. *et al.* (2012) 'Associated computational plasticity schemes for nonassociated frictional materials', *International Journal for Numerical Methods in Engineering*, 90(9), pp. 1089–1117.

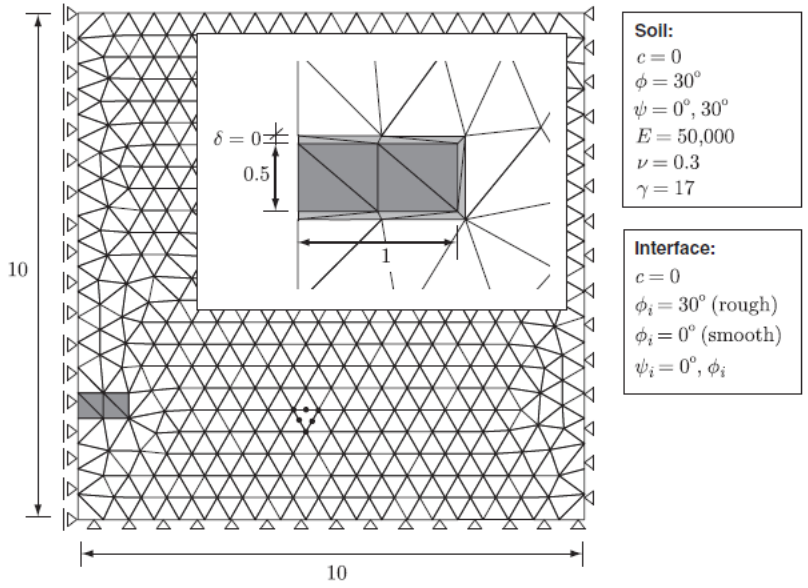


Figure 17. Anchor in purely frictional soil: problem setup and finite element mesh (coarse).

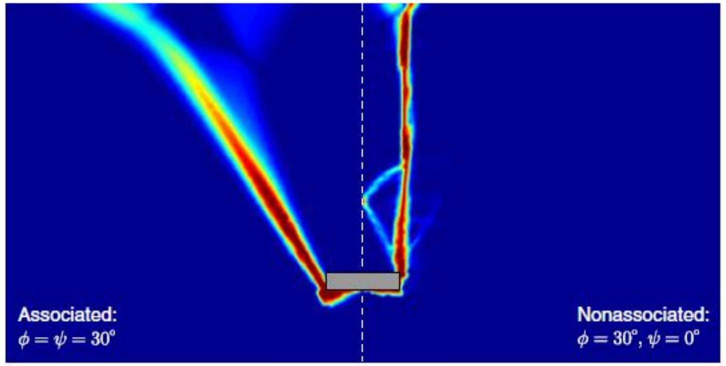
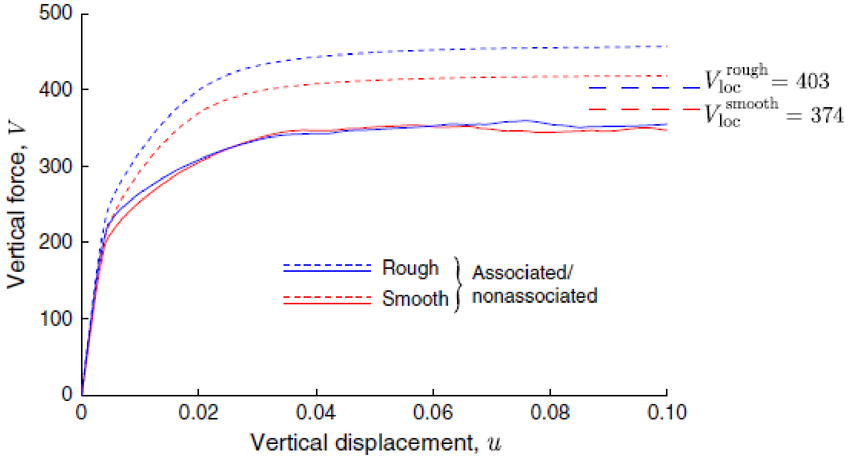


Figure 19. Anchor problem: rates of plastic shear strain at collapse.



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c-φ-Reduction Example in ZSoil

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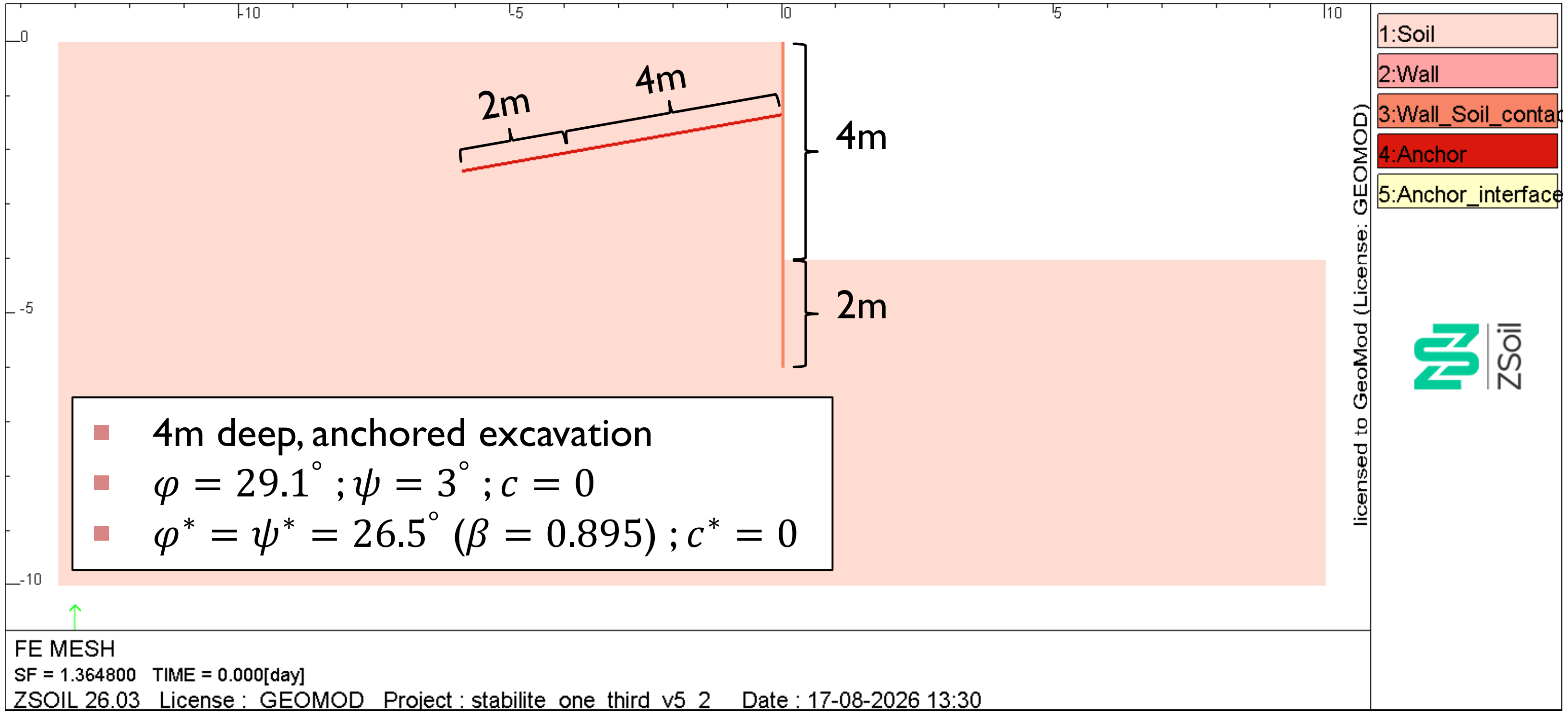
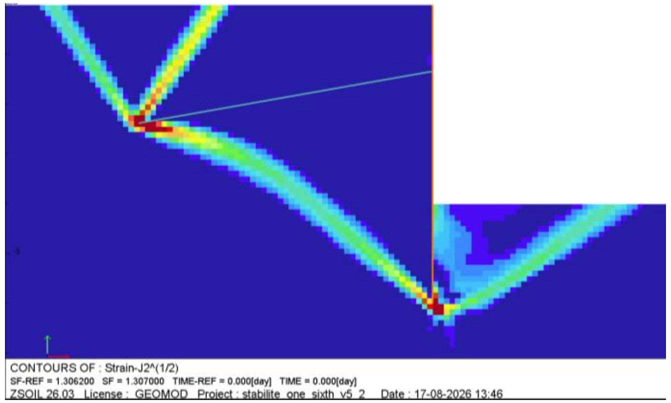
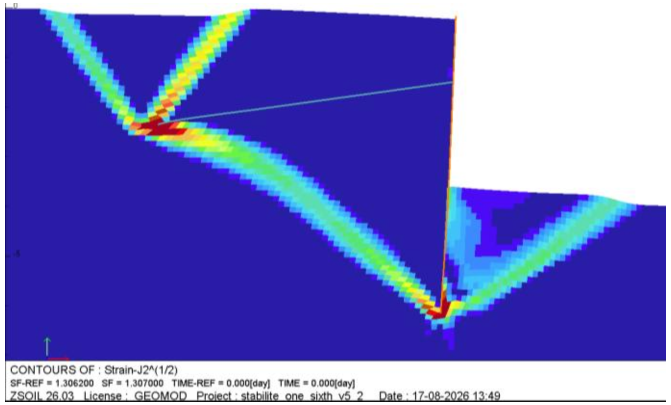


c-φ-reduction
failure mechanism:





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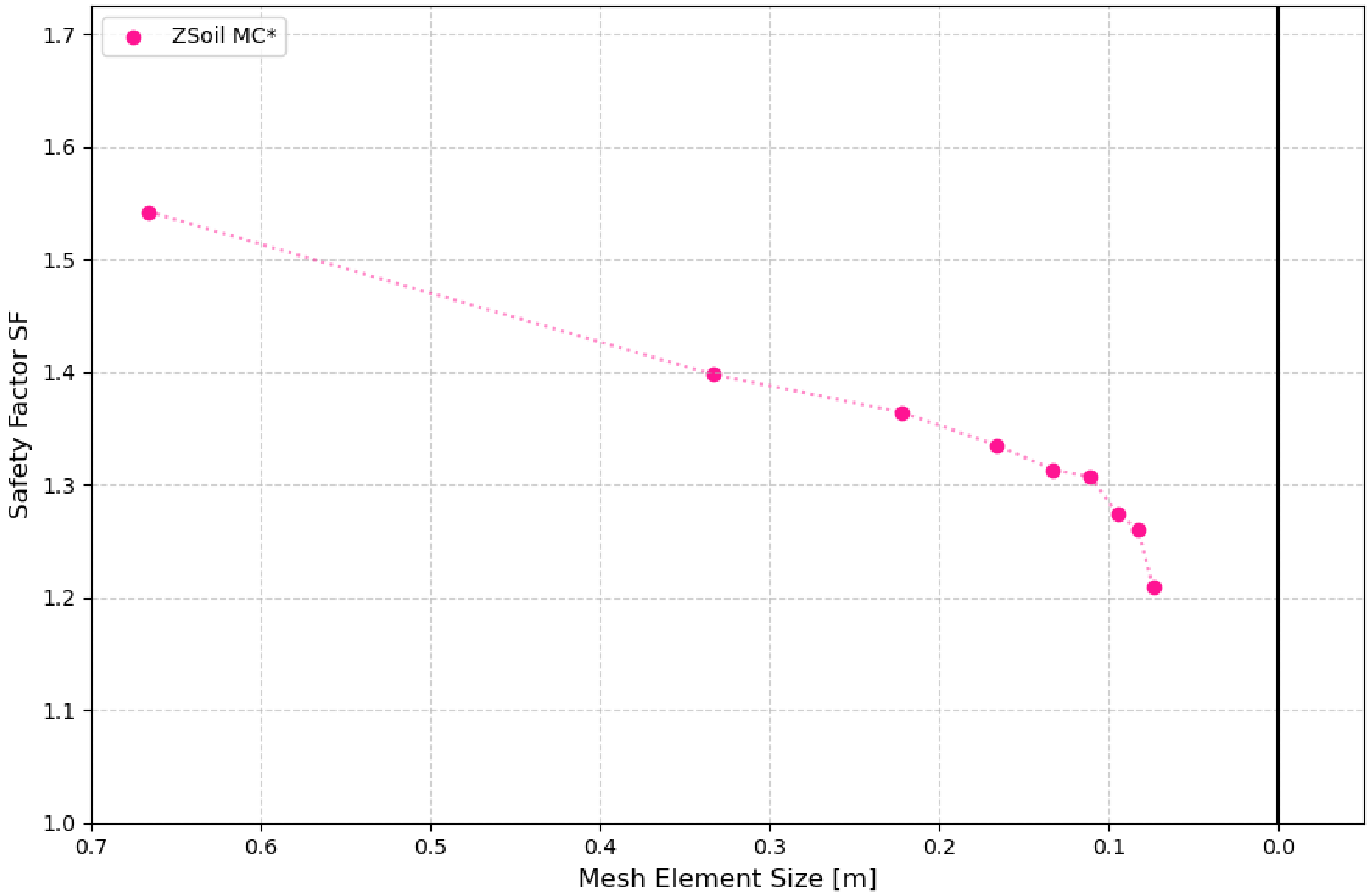

- MC: Lin. el. + **M**ohr-**C**oulomb failure
- *: Davies' reduced strength parameters with AFR
- AFR: $\psi = \varphi$; NAFR: $\psi \neq \varphi$

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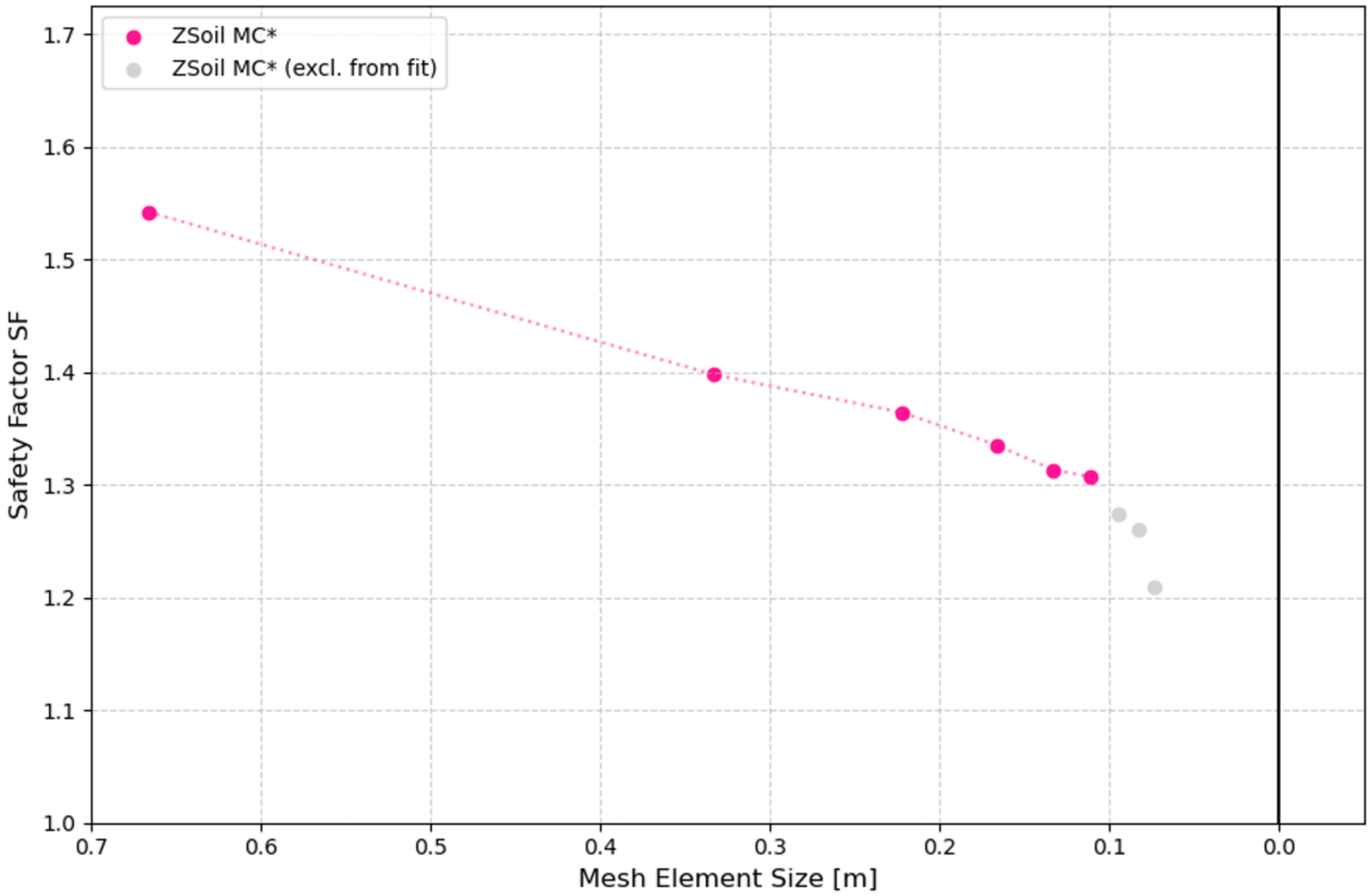

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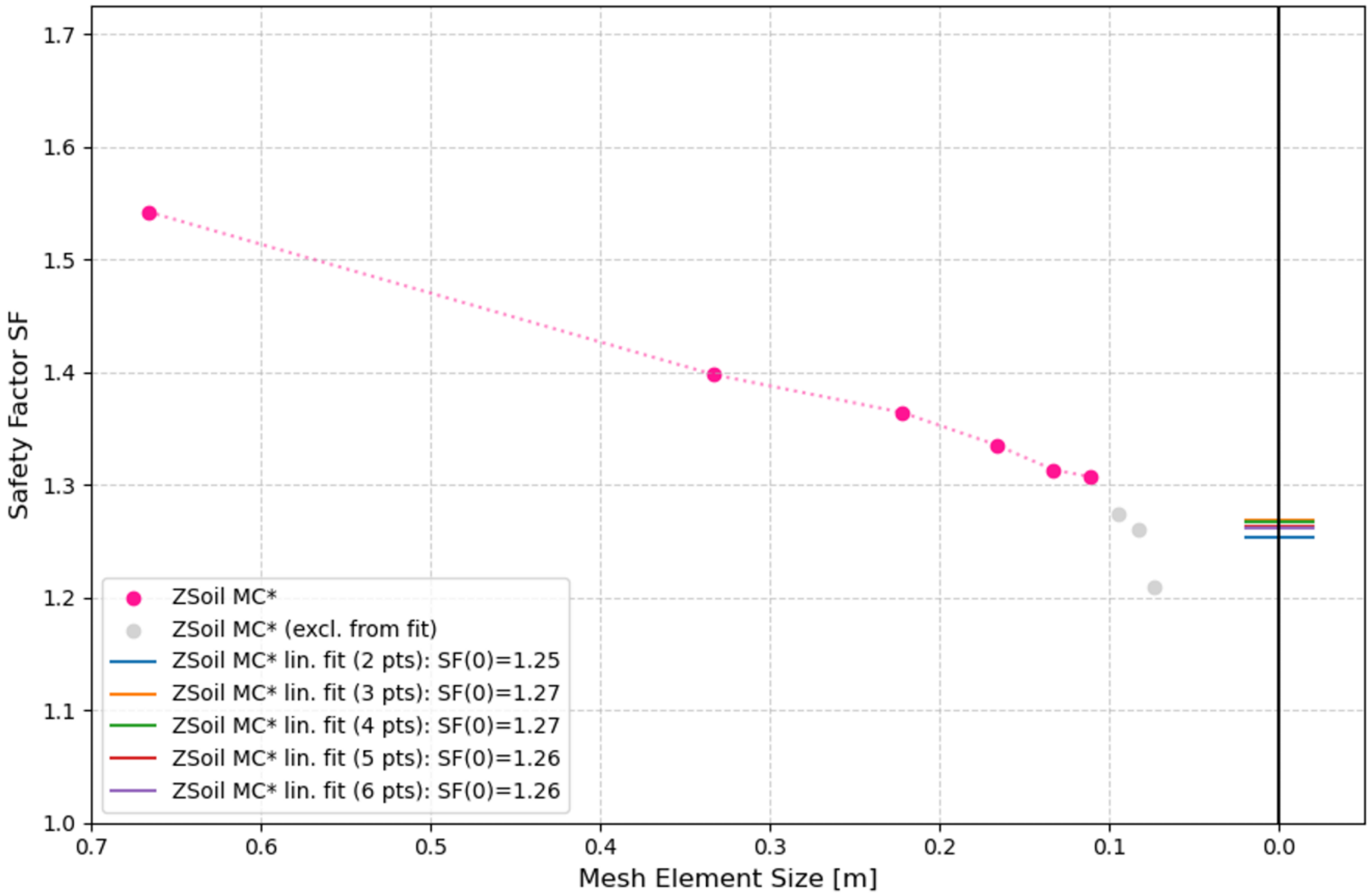

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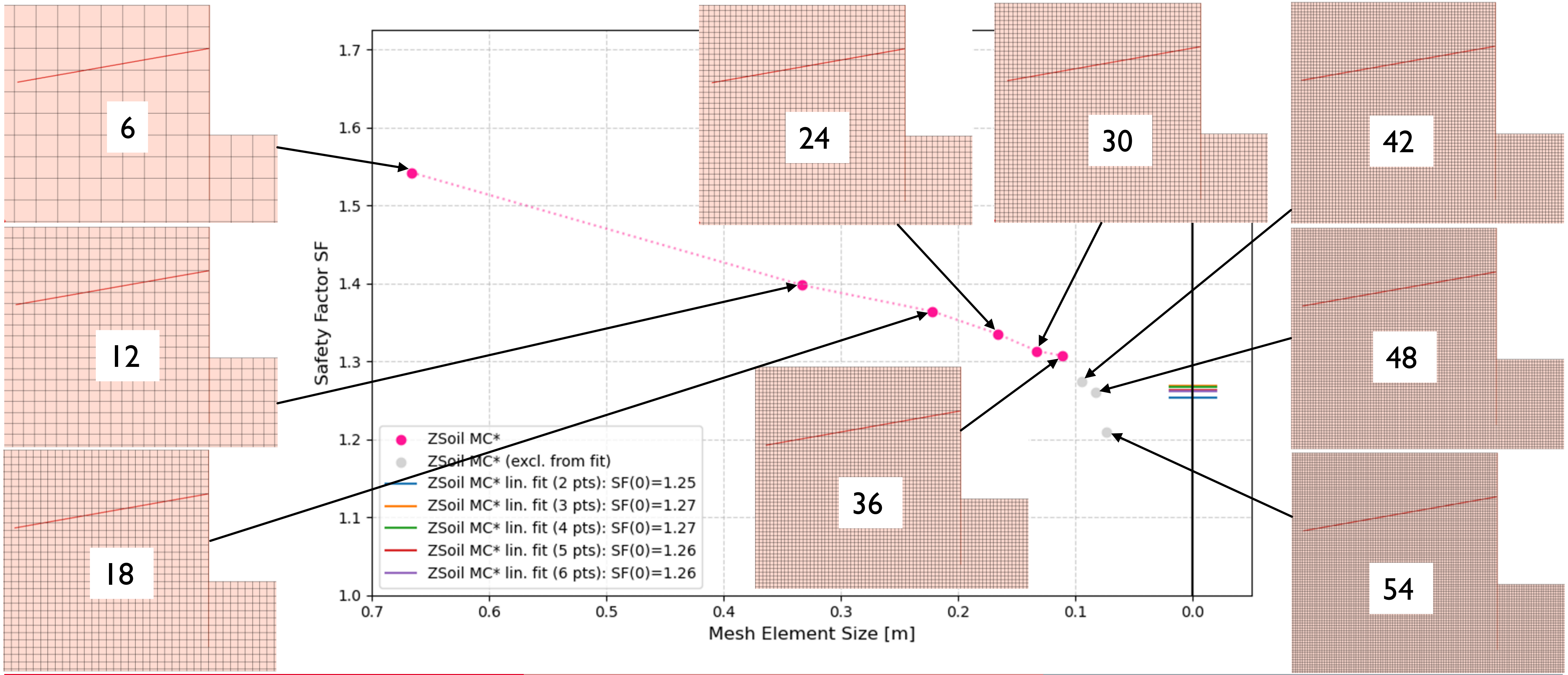

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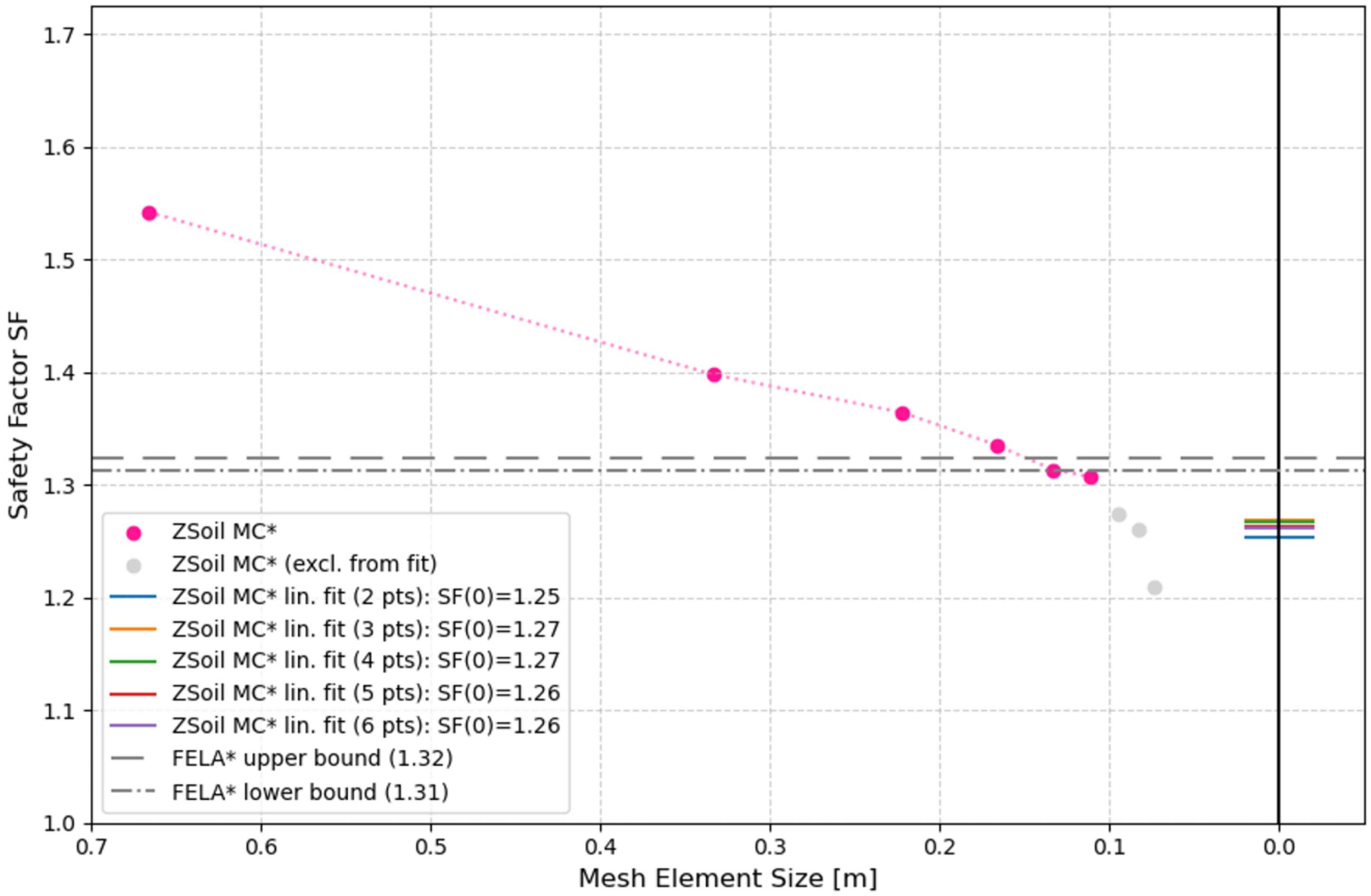

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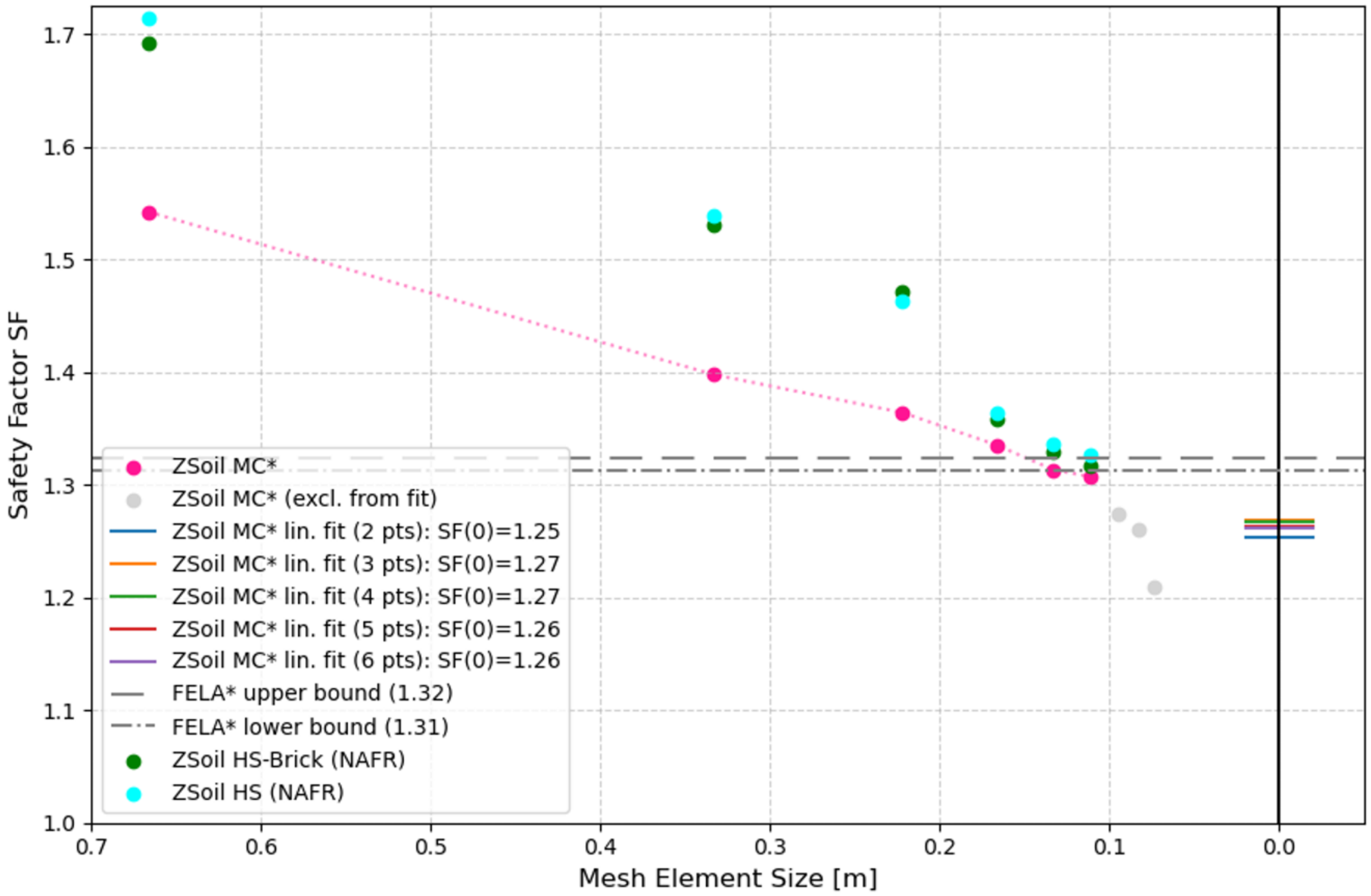

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- HS-Brick: HS with small strain extension (*Brick* version)

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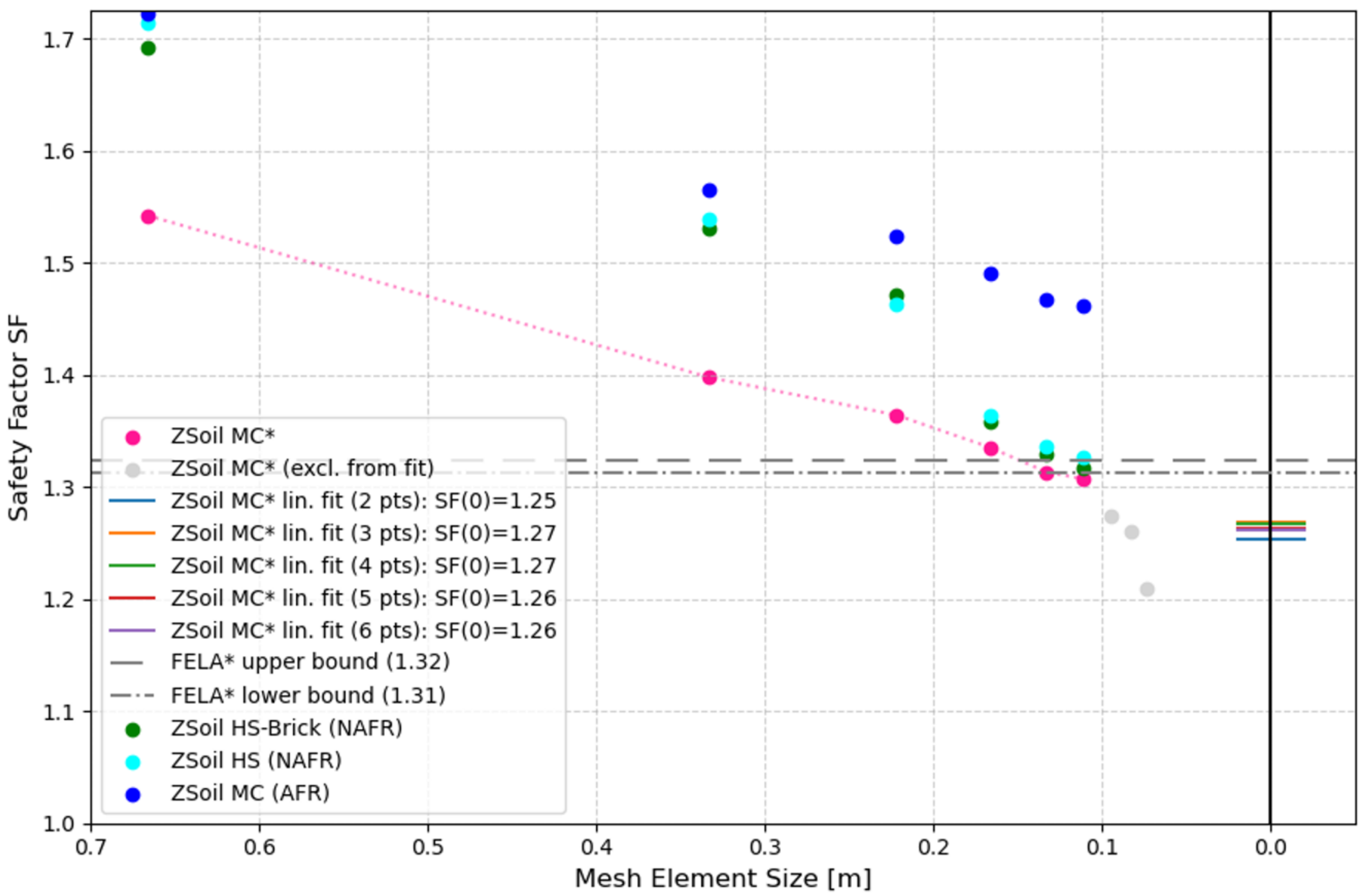

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